St Modan’s High School

Parental Numeracy Booklet

What does it mean to be numerate?

Being numerate helps us to function responsibly in everyday life and contribute effectively to society. It increases our opportunities within the world of work and establishes foundations which can be built upon through lifelong learning. Numeracy is not only a subset of mathematics; it is also a life skill which permeates and supports all areas of learning, allowing young people access to the wider curriculum.

We are numerate if we have developed:

> *the confidence and competence in using number which will allow individuals to solve problems, analyse information and make informed decisions based on calculations*.

A numerate person will have acquired and developed fundamental skills and be able to carry out number processes but, beyond this, being numerate also allows us to access and interpret information, identify possibilities, weigh up different options and decide on which option is most appropriate.

Numeracy is a skill for life, learning and work. Having well-developed numeracy skills allows young people to be more confident in social settings and enhances enjoyment in a large number of leisure activities. For these and many other reasons, all teachers have important parts to play in enhancing the numeracy skills of all children and young people.

Numerate people rely on the accumulation of knowledge, concepts and skills they have developed, and continually revisit and add to these. All practitioners, as they make use of the statements of experiences and outcomes to plan learning, will ensure that the numeracy skills developed from early levels and beyond are revisited and refreshed throughout schooling and into lifelong learning.

Numeracy Across Learning
Principles and Practice

Purpose of this booklet…

This booklet has been created for the parents/carers of pupils attending St Modan's High School. It outlines *some* of the methods that we use in Maths and across other subjects for teaching the most important Numeracy skills.

The booklet has been created by a group of teachers from across a variety of subject areas and reflect the learning that pupils will experience through their Broad General Education and beyond.

The key purpose of the booklet is to allow parents/carers the opportunity to see how Numeracy has been taught and give you the tools required to help your son/daughter to improve their Numeracy skills.

Contents

Estimation and Rounding	**3**
Rounding numbers	4
Significant figures	7
Estimation	10
Number and Number Processes	**11**
Addition	12
Subtraction	14
Multiplication	16
Division	19
Integers – Adding/Subtracting	22
Integers – Multiplying/Dividing	25
Order of Operations (BODMAS)	28
Fractions Decimal Fractions and Percentages	**30**
Fractions	31
Simplifying	31
Mixed and Improper fractions	33
Multiplying	35
Dividing	36
Adding/Subtracting	37
Percentages	39
Money	**42**
Best Buy	43
Foreign Exchange	46
Time	**48**
Time Differences	49
Fractional and Decimal Time	51
Speed Distance and Time	53
Measurement	**56**
Length and Weight	57
Area and Volume	58
Area and Volume Calculations	61
Data and Analysis	**64**
Averages	65
Graphical Displays	68
Ideas of Chance and Uncertainty	**75**
Probability and Chance	76
Probability Calculations	76
Solutions to Practice Questions	**79**

Estimation and Rounding

	Benchmarks
Second Level	• Rounds whole numbers to the nearest 1000, 10 000 and 100 000. • Rounds decimal fractions to the nearest whole number, to one decimal place and two decimal places. • Applies knowledge of rounding to give an estimate to a calculation appropriate to the context.
Third Level	• Rounds decimal fractions to three decimal places. • Uses rounding to routinely estimate the answers to calculations.
Fourth Level	• Rounds answers to a specified significant figure. • Demonstrates that the context of the question needs to be considered when rounding. • Demonstrates the impact of inaccuracy and error, for example, the impact of rounding an answer before the final step in a multi-step calculation. • Uses a given tolerance to decide if there is an allowable amount of variation of a specified quantity, for example, dimensions of a machine part, 235 mm ± 1 mm.

Rounding numbers

Numbers are rounded off at times where exact detail is not totally necessary.

Simple examples:

- A pair of jeans costing £24.99 would usually be rounded off to £25 or
- The volume of a bottle as 2123ml would normally be rounded off to 2100ml or even 2000ml.

The rule for this rounding relies on our knowledge of the decimal columns: Units, Tens, Hundreds, Thousands and so on. This also extends to columns smaller than units: tenths, hundredths, thousandths and so on.

Th	H	T	U	.	t	h	th

The numbers from before would look like…

T	U	.	t	h
2	4	.	9	9

Th	H	T	U
2	1	2	3

To round these numbers we need to know how much detail is required; we could be asked to round to the nearest Unit, Ten, Hundred etc or to a specific number of decimal places, usually 1dp or 2dp.

We use the column to the right of this requirement to help us do the rounding using the following rule:

If the column to the right is 0, 1, 2, 3 or 4 then the number is not changed.
If the column to the right is 5, 6, 7, 8 or 9 then the number is rounded up.

Stays the same					Rounds up				
0	1	2	3	4	5	6	7	8	9

Example 1:
Round the number 235 to the nearest 10.

We are interested in the *Tens* column, we can see the next column is the *Units*, we use the rule above to help us round. This will make the *Tens* column round up and therefore the number rounds to 240.

H	T	U
2	3	5

Example 2:
Round the number 35.346 to 1dp.

We are interested in the *tenths* column, we can see the next column is the *hundredths*, we use the rule above to help us round.

T	U	.	t	h	th
3	5	.	3	4	6

This will make the *tenths* column remain the same and therefore the number rounds to 35.3.

Practice Questions:

1. Round to the nearest unit:

(a)	2·9	**(b)**	5·6	**(c)**	1·4	**(d)**	8·3

2. Round to the nearest unit:

(a)	12·4	**(b)**	35·13	**(c)**	27·6	**(d)**	82·711

3. Round to the nearest ten:

(a)	23	**(b)**	74	**(c)**	68	**(d)**	85

4. Round to the nearest ten:

(a)	213	**(b)**	1281	**(c)**	7616	**(d)**	344

5. Round to the nearest hundred:

(a)	270	**(b)**	150	**(c)**	3405	**(d)**	83063

6. Round the following numbers to 1 decimal place:

(a)	0·31	**(b)**	0·29	**(c)**	0·56	**(d)**	0·61

7. Round the following numbers to 1 decimal place:

(a)	2·91	**(b)**	5·681	**(c)**	1·475	**(d)**	8·33

8. Round the following numbers to 2 decimal places:

(a)	62·035	**(b)**	15·619	**(c)**	31·475	**d)**	18·303

9. Round the following numbers to 2 decimal places:

(a)	2·915	**(b)**	5·663	**(c)**	1·408	**(d)**	8·321

Significant figures

Rounding to significant figures is similar to normal rounding, but this time to a specific number of *most important* digits.

Simple examples:

- A lottery win of £9 706 000 would normally be rounded to £10 000 000
- The attendance at a football match of 29 023 would normally be rounded to 29 000 or even 30 000.

The rule for this rounding relies on our knowledge that the numbers that appear first are the most important (*most significant*); in this we would be looking for the first non-zero as the 1st significant digit and the other significant digits follow immediately after this.

The numbers from before would look like…

9	7	0	6	0	0	0
1st	2nd	3rd	4th	5th	6th	7th

2	9	0	2	3
1st	2nd	3rd	4th	5th

To round these numbers we need to know how much detail is required; we could be asked to round to the 1sf, 2sf, 3sf etc.

We use the column to the right of this requirement to help us do the rounding using the following rule:

If the column to the right is 0, 1, 2, 3 or 4 then the number is not changed.
If the column to the right is 5, 6, 7, 8 or 9 then the number is rounded up.

Stays the same					Rounds up				
0	1	2	3	4	5	6	7	8	9

Example 1:
Round the number 2315 to 3 significant figures.

We are interested in the *3rd significant figure*, we use the *4th significant figure* to help us round.

2	3	1	5
1st	2nd	3rd	4th

This will make the *3rd significant figure* round up and therefore the number rounds to 2320.

Example 2:
Round the number 0.0003524 to 2 significant figures.

We are interested in the 2nd *significant figure*, we use the 3rd *significant figure* to help us round.

0	.	0	0	0	3	5	2	4
					1st	2nd	3rd	4th

This will make the 2nd *significant figure* remain the same and therefore the number rounds to 0.00035.

Practice Questions:

1. Round to 1 significant figure:

(a)	23	(b)	5·5	(c)	78	(d)	31
(e)	125	(f)	309	(g)	291	(h)	843.6

2. Round to 2 significant figures:

(a)	10·9	(b)	556·2	(c)	3·98	(d)	12345
(e)	1·01	(f)	93	(g)	55·86	(h)	606

3. Round to 1 significant figure:

(a)	8·72	(b)	92·8	(c)	0·186	(d)	679
(e)	2·112	(f)	6·463	(g)	31·4	(h)	25·8

4. Round to 2 significant figures:

(a)	24·27	(b)	18·76	(c)	6397	(d)	4·99
(e)	0·0526	(f)	0·00613	(g)	0·08702	(h)	13814

Estimation

We can use rounded numbers to make estimates; this can be useful in checking that a calculation makes sense. That is, using an estimated answer as a guide.

Simple example:

- In calculating 22×38 we could calculate 20×40 initially (it's easier!), meaning I know our answer should be around 800.

Example 1:
Raffle tickets are sold at an event, the information on the sales is shown in the table below:

Colour	Pink	White	Green	Blue
Number Sold	27	44	38	33

We can estimate the number sold: $30 + 40 + 40 + 30$
$= 140$

Now the actual answer we get: $27 + 44 + 38 + 33$
$= 142$

This answer is around 140 and therefore we consider this to be correct.

Example 2:
Calculate the answer to 54×27.

First the estimate is 50×30; the answer is around 1500.

Now the actual answer we get for 54×27 is 1258, this is **not** around 1500 and therefore we consider this to be incorrect.

At this point we would re-evaluate and try to fix our mistake.

The new answer we get is 1458, this is around 1500 and therefore we consider this to now be correct.

*****The methods used to get the correct answers above are covered in the next section*****

Number and Number Processes

	Benchmarks
Second Level	• Reads, writes and orders whole numbers to 1 000 000, starting from any number in the sequence. • Explains the link between a digit, its place and its value for whole numbers to 1 000 000. • Reads, writes and orders sets of decimal fractions to three decimal places. • Explains the link between a digit, its place and its value for numbers to three decimal places. • Partitions a wide range of whole numbers and decimal fractions to three decimal places, for example, 3·6 = 3 ones and 6 tenths = 36 tenths. • Adds and subtracts multiples of 10, 100 and 1000 to and from whole numbers and decimal fractions to two decimal places. • Adds and subtracts whole numbers and decimal fractions to two decimal places, within the number range 0 to 1 000 000. • Uses multiplication and division facts to the 10^{th} multiplication table. • Multiplies and divides whole numbers by multiples of 10, 100 and 1000. • Multiplies and divides decimal fractions to two decimal places by 10, 100 and 1000. • Multiplies whole numbers by two digit numbers. • Multiplies decimal fractions to two decimal places by a single digit. • Divides whole numbers and decimal fractions to two decimal places, by a single digit, including answers expressed as decimal fractions, for example, 43 ÷ 5 = 8·6. • Identifies familiar contexts in which negative numbers are used. • Orders numbers less than zero and locates them on a number line.

Third Level	• Recalls quickly multiplication and division facts to the 10^{th} multiplication table. • Uses multiplication and division facts to the 12^{th} multiplication table. • Solves addition and subtraction problems working with whole numbers and decimal fractions to three decimal places. • Solves addition and subtraction problems working with integers. • Solves multiplication and division problems working with whole numbers and decimal fractions to three decimal places. • Solves multiplication and division problems working with integers.

Fourth Level	• Interprets and solves multi-step problems using the four operations.

Addition

There are many strategies for adding numbers together, at St Modan's we approach this in a model that provides full understanding. In the first instance, we do not teach addition through columns – though this may be used at a later stage (once understanding is sound).

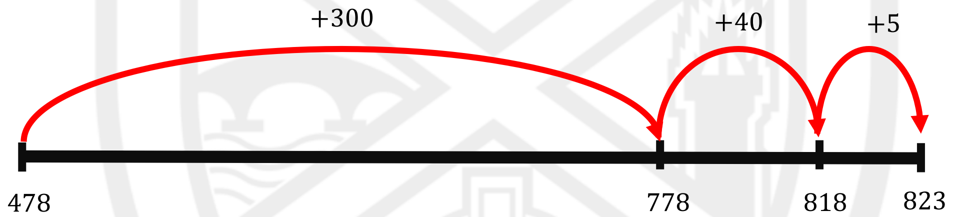
We use an *empty number line* to complete additions; this works by extending our basic principle of counting on in the very early stages of Numeracy education.

Example 1:
Find the answer to $478 + 345$.

We start at 478 and *jump* up in increments that we would know the answer to. One example of these jumps is shown below;

As you will see we finish at 823, our answer to the calculation.

We can also end up with the same answer by taking different steps, as below.



Example 2:
Find the answer to $3.28 + 1.34$.

We start at 3.28 and *jump* up in increments that we would know the answer to. One example of these jumps is shown below;

As you will see we finish at 4.62, our answer to the calculation.

Practice Questions:

1	**(a)**	52	+	61	**(b)**	76	+	68	**(c)**	93	+	91	**(d)**	85	+	149
	(e)	79	+	115	**(f)**	86	+	81	**(g)**	64	+	273	**(h)**	70	+	195
2	**(a)**	135	+	86	**(b)**	81	+	63	**(c)**	115	+	77	**(d)**	55	+	127
	(e)	61	+	108	**(f)**	129	+	114	**(g)**	102	+	169	**(h)**	84	+	162
3	**(a)**	172	+	68	**(b)**	249	+	74	**(c)**	297	+	65	**(d)**	271	+	149
	(e)	276	+	93	**(f)**	157	+	102	**(g)**	179	+	266	**(h)**	306	+	212
4	**(a)**	9.2	+	7.8	**(b)**	8	+	9.4	**(c)**	5.5	+	8.1	**(d)**	5	+	7.5
	(e)	9.1	+	10	**(f)**	7.2	+	14.1	**(g)**	5.7	+	22.5	**(h)**	7.7	+	15.1
5	**(a)**	12.7	+	8.1	**(b)**	13.7	+	6.1	**(c)**	13.4	+	5.2	**(d)**	11.2	+	8.8
	(e)	14.6	+	5.7	**(f)**	13	+	5.6	**(g)**	14	+	31.7	**(h)**	9.9	+	32.4
6	**(a)**	17	+	5.8	**(b)**	28.8	+	7	**(c)**	18.8	+	9.9	**(d)**	26.3	+	9.5
	(e)	16.7	+	8.2	**(f)**	25	+	7.8	**(g)**	18.7	+	33.1	**(h)**	33.5	+	32.1

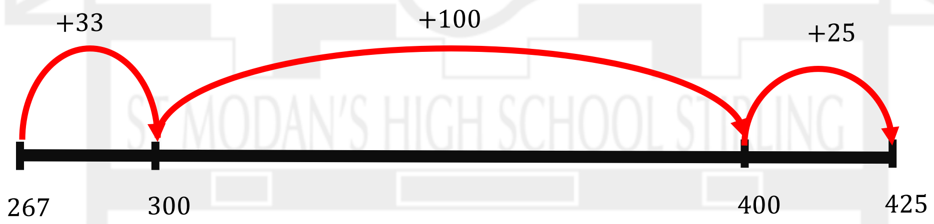
Subtraction

There are also many strategies for subtracting numbers together, at St Modan's we approach this in a model that again provides full understanding. In the first instance, we do not teach subtraction through columns – though this may be used at a later stage (once understanding is sound).

We use an *empty number line* from before to complete subtractions.

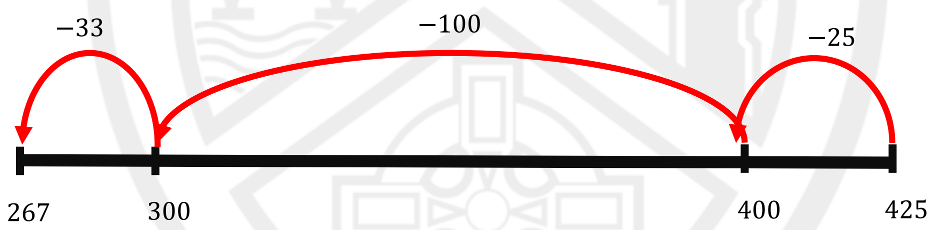
Example 1:
Find the answer to $425 - 267$.

We could start at 267 and *jump* up in increments that we would know the answer to. One example of these jumps is shown below:



As you will see we have jumped a total of 158, our answer to the calculation. We have thought about this subtraction as a *difference*.

We can also end up with the same answer by taking different steps, as below, starting at 425.



We have taken away a total of 158, our answer to the calculation.

Example 2:
Find the answer to $3.48 - 1.34$.

We start at 1.34 and *jump* up in increments that we would know the answer to. One example of these jumps is shown below;

As you will see we have jumped a total of 2.14, our answer to the calculation. We have thought about this subtraction as a *difference*.

Practice Questions:

1	**(a)**	95	-	76	**(b)**	68	-	60	**(c)**	58	-	58	**(d)**	102	-	68
	(e)	119	-	58	**(f)**	125	-	62	**(g)**	197	-	58	**(h)**	235	-	84
2	**(a)**	123	-	95	**(b)**	190	-	141	**(c)**	53	-	52	**(d)**	287	-	144
	(e)	192	-	108	**(f)**	103	-	72	**(g)**	281	-	119	**(h)**	272	-	53
3	**(a)**	412	-	297	**(b)**	452	-	338	**(c)**	586	-	328	**(d)**	423	-	271
	(e)	589	-	294	**(f)**	359	-	265	**(g)**	332	-	270	**(h)**	385	-	224
4	**(a)**	9.6	-	7.4	**(b)**	6.9	-	5.8	**(c)**	16	-	8.3	**(d)**	17	-	8.8
	(e)	11	-	5.4	**(f)**	12	-	5.1	**(g)**	15	-	7	**(h)**	21	-	8.9
5	**(a)**	5.6	-	5.4	**(b)**	18	-	15	**(c)**	20	-	10	**(d)**	18	-	8.3
	(e)	16	-	15	**(f)**	27	-	13	**(g)**	24	-	14	**(h)**	20	-	7.5
6	**(a)**	46	-	32	**(b)**	49	-	32	**(c)**	28	-	20	**(d)**	54	-	34
	(e)	13	-	9.8	**(f)**	56	-	24	**(g)**	72	-	33	**(h)**	29	-	23

Multiplication

Multiplication is primarily taught using a method that is a common mental strategy, thus allowing a better understanding of the calculation taking place. In the first instance, we do not teach multiplication through columns – though this may be used at a later stage (once understanding is sound).

Example 1:

Find the answer to 23×7.

When we do this mentally, most people will calculate 20×7 and 3×7 and add the answers together. This is our basic understanding and intuition; therefore we also write it in a similar format:

$$23 \times 7$$

$$20 \times 7 = 140 \qquad 3 \times 7 = 21$$

$$140 + 21 = 161$$

This can be shortened to a more organised model (an array) as follows.

	20	**3**
7	140	21

$$140 + 21 = 161$$

Example 2:
Find the answer to 36×47.

This can be completed using our new organised method (array).

	30	**6**
40	1200	240
7	210	42

$$1200 + 210 = 1410 \qquad 240 + 42 = 282$$

$$1410 + 282 = 1692$$

Therefore $36 \times 47 = 1692$.

This method can also be extended to multiplying numbers with decimal places.

Example 3:
Find the answer to 4.3×3.2.

This can be completed using our new organised method (array), but initially ignoring the point. 4.3 is treated as 43 and 3.2 is treated as 32.

	40	**3**
30	1200	90
2	80	6

$$1200 + 80 = 1280 \qquad 90 + 6 = 96$$

$$1280 + 96 = 1376$$

$43 \times 32 = 1376$, however this was not the initial question.
As we ignored two decimal places in the first instance (one from either number), we can now put them back in to obtain the correct result.

Therefore, $4.3 \times 3.2 = 13.76$.

Practice Questions:

1	**(a)**	65	x	4	**(b)**	87	x	8	**(c)**	96	x	9	**(d)**	90	x	9
	(e)	98	x	2	**(f)**	71	x	8	**(g)**	70	x	8	**(h)**	70	x	5
2	**(a)**	78	x	75	**(b)**	78	x	58	**(c)**	53	x	96	**(d)**	84	x	96
	(e)	72	x	96	**(f)**	97	x	74	**(g)**	56	x	60	**(h)**	61	x	96
3	**(a)**	93	x	53	**(b)**	473	x	67	**(c)**	871	x	68	**(d)**	221	x	85
	(e)	92	x	55	**(f)**	337	x	75	**(g)**	39	x	93	**(h)**	467	x	62
4	**(a)**	8.1	x	1	**(b)**	5.1	x	0.6	**(c)**	9.7	x	1	**(d)**	7	x	0.6
	(e)	7.9	x	0.7	**(f)**	7.2	x	0.3	**(g)**	6	x	0.6	**(h)**	5.2	x	1
5	**(a)**	7.8	x	7.7	**(b)**	5.9	x	5.9	**(c)**	7.2	x	9.8	**(d)**	5.1	x	9
	(e)	7.3	x	7.3	**(f)**	6.2	x	9.4	**(g)**	8.3	x	7.8	**(h)**	6	x	6
6	**(a)**	1.83	x	5.1	**(b)**	5.67	x	5.3	**(c)**	6.09	x	9.5	**(d)**	1.69	x	9.3
	(e)	9.2	x	8.9	**(f)**	0.27	x	7.4	**(g)**	2.47	x	7.5	**(h)**	6.35	x	8.5

Division

Division in its simplest sense the *sharing* of quantities. At the earliest stage of learning pupils would be asked how to share 10 sweets between 2 people. This basic concept can be extended to more difficult numbers. The method below is called the *bar method*. At later stages, where understanding is solid we can move on to the more traditional methods of division.

Example 1:
Complete the following calculation $48 \div 4$

Firstly, we create a 'bar' for the number 48 and a bar that splits this in 4 equal parts.

48			
?	?	?	?

Now we ask what number should be in each part, this can be done by recalling multiplication facts or using an informed 'guess'. Let's say I think the answer is 10.

48				
10	10	10	10	$= 40$

Using addition/multiplication I will see that I fall short of the correct total, I now know 10 is not enough – now I try to improve on this. I might now try 13.

48				
13	13	13	13	$= 52$

Now my total is too high… I will now try 12.

48				
12	12	12	12	$= 48$

This is now the correct total, so I can say that $48 \div 4 = 12$.

This method of making a good guess and approaching the correct answer works very well for smaller divisors (the number we divide by), and also works very well for numbers with decimal places. However when we have larger numbers we might find it somewhat of a task to write it all out, we can do it in the following way.

Example 2:
Complete the following calculation $1058 \div 23$

Firstly, we create a 'bar' for the number 1058 and a bar that splits this in 23 equal parts (but without fully doing so).

1058			
?	?	…	?

Now we ask what number should be in each part, this can be done by recalling multiplication facts or using an informed 'guess'. Let's say I think the answer is 20.

1058			
20	20	…	20

$= 23 \times 20 = 460$

Using my multiplication skills from earlier I will see that I fall short of the correct total, I now know 20 is not enough – now I try to improve on this. I might now try 40.

1058			
40	40	…	40

$= 23 \times 40 = 920$

Now my total is still below, but better at least! I continue my process of improvement until I get to the correct total.

1058			
46	46	…	46

$= 23 \times 46 = 1058$

This is now the correct total, so I can say that $1058 \div 23 = 46$.

You will see that this relies heavily on understanding multiplication. Giving a further understanding and recognition that division is simply the opposite of multiplication.

Practice Questions:

1	**(a)**	15	÷	3	**(b)**	18	÷	6	**(c)**	231	÷	11	**(d)**	27	÷	9
	(e)	10	÷	10	**(f)**	300	÷	10	**(g)**	189	÷	7	**(h)**	46	÷	2
2	**(a)**	270	÷	18	**(b)**	195	÷	13	**(c)**	208	÷	13	**(d)**	464	÷	16
	(e)	170	÷	10	**(f)**	432	÷	16	**(g)**	276	÷	12	**(h)**	270	÷	18
4	**(a)**	12.6	÷	6	**(b)**	4.8	÷	3	**(c)**	20	÷	10	**(d)**	17	÷	6
	(e)	6	÷	4	**(f)**	3	÷	2	**(g)**	27	÷	10	**(h)**	6	÷	5
5	**(a)**	1.62	÷	9	**(b)**	0.6	÷	6	**(c)**	1.1	÷	6	**(d)**	1	÷	10
	(e)	1.4	÷	7	**(f)**	1	÷	9	**(g)**	1.4	÷	5	**(h)**	0.6	÷	4

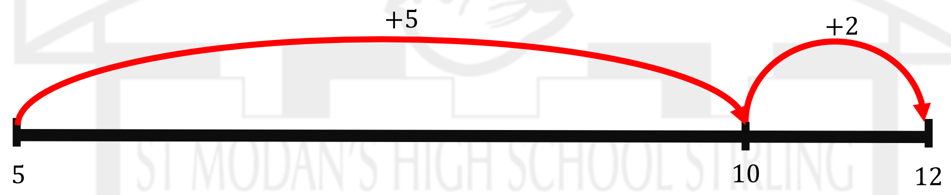
Integers – Adding/Subtracting

Integers is the collective name for all numbers which can be written without a fraction/decimal/square root etc. That is the simple counting numbers which extend from negative numbers through zero to positive numbers. A mercury filled thermometer for example generally uses these numbers.

Here we look at the rules that relate to adding, subtracting, multiplying and dividing integers.

Example 1:
Calculate $5 + 7$

From before we might put this in a number line, moving 7 to the right:



Calculate $5 - 7$

If we think about $+7$ and -7 as the opposite of one another then this says to do the opposite of moving 7 to the right (move 7 to the left).

Adding goes to the right, subtracting goes to the left.

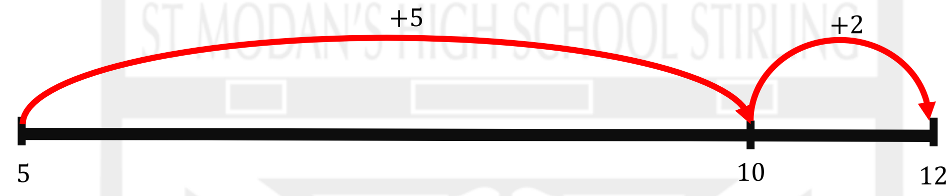
The calculations above are generally well understood and intuitive; we move to the right when we add, we move to the left when we subtract. The area that pupils find more challenging are those that follow in example 2.

Example 2:
Calculate $5 + (-7)$

In this we are adding a move to the left!

Calculate $5 - (-7)$

In this we are subtracting a move to the left – therefore moving to the right!



Remember, subtracting is the instruction opposite to adding.

Practice Questions:

1	**(a)**	10	+	8	**(b)**	14	+	12	**(c)**	11	+	7	**(d)**	12	+	11
	(e)	7	+	11	**(f)**	6	+	13	**(g)**	12	+	14	**(h)**	7	+	6
2	**(a)**	9	-	7	**(b)**	6	-	12	**(c)**	10	-	6	**(d)**	7	-	10
	(e)	15	-	7	**(f)**	12	-	7	**(g)**	14	-	8	**(h)**	13	-	11
3	**(a)**	-19	+	10	**(b)**	-10	+	10	**(c)**	-19	+	5	**(d)**	-14	+	15
	(e)	-11	-	14	**(f)**	-16	-	9	**(g)**	-13	-	5	**(h)**	-11	-	8
4	**(a)**	12	+	(-17)	**(b)**	15	+	(-13)	**(c)**	11	+	(-12)	**(d)**	12	+	(-19)
	(e)	-14	+	(-18)	**(f)**	-19	+	(-10)	**(g)**	-15	+	(-11)	**(h)**	-13	+	(-14)

Integers – Multiplying/Dividing

Multiplying and dividing integers (and other numbers) follows the same set of rules. Understanding how these rules work can be illustrated as follows.

Using a standard multiplication grid we can see that horizontal and vertical sequences are created.

x	**3**	**2**	**1**	**0**
3	9	6	3	0
2	6	4	2	0
1	3	2	1	0
0	0	0	0	0

We can also see that the results are as expected; positive numbers multiplied by positive numbers generate a positive result.

$+ \times + = +$

If we extend the table and continue the sequences, we can start to generate further rules…

x	**3**	**2**	**1**	**0**
3				
2				
1				
0				
-1				
-2				
-3				

x	**3**	**2**	**1**	**0**	**-1**	**-2**	**-3**
3							
2							
1							
0							

We can see that the positive numbers, when multiplied by negative numbers, produce a negative result.

$+ \times - = -$ and $- \times + = -$

x	3	2	1	0	-1	-2	-3
3	9	6	3	0	[illegible]	[illegible]	[illegible]
2	6	4	2	0	[illegible]	[illegible]	[illegible]
1	3	2	1	0	[illegible]	[illegible]	[illegible]
0	0	0	0	0	[illegible]	[illegible]	[illegible]
-1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Continuing the patterns further we can see that the negative numbers, when multiplied by negative numbers, produce a positive result.

$$- \times - = +$$

The rules that apply for multiplying, also apply for dividing.

$+ \times + = +$	$+ \div + = +$
$+ \times - = -$	$+ \div - = -$
$- \times + = -$	$- \div + = -$
$- \times - = +$	$- \div - = +$

In general, when the signs are the same we get a positive result, when they are different we get a negative result.

Example 1:
Calculate each of the following:

(a) 7×5
$= 35$

(b) -8×6
$= -48$

(c) $5 \times (-6)$
$= -30$

(d) $-8 \times (-3)$
$= 24$

Example 2:
Calculate each of the following:

(a) $42 \div 6$
$= 7$

(b) $-30 \div 5$
$= -6$

(c) $55 \div (-11)$
$= -5$

(d) $-40 \div (-10)$
$= 4$

Practice Questions:

1	**(a)**	2	x	8	**(b)**	7	x	11	**(c)**	5	x	2	**(d)**	11	x	4
	(e)	2	x	(-3)	**(f)**	2	x	(-7)	**(g)**	12	x	(-9)	**(h)**	9	x	(-4)
2	**(a)**	-4	x	9	**(b)**	-9	x	6	**(c)**	-1	x	7	**(d)**	-7	x	3
	(e)	-9	x	(-3)	**(f)**	-1	x	(-3)	**(g)**	-3	x	(-3)	**(h)**	-9	x	(-1)
4	**(a)**	120	÷	10	**(b)**	21	÷	3	**(c)**	132	÷	11	**(d)**	20	÷	10
	(e)	48	÷	(-4)	**(f)**	27	÷	(-3)	**(g)**	35	÷	(-5)	**(h)**	22	÷	(-2)
5	**(a)**	-15	÷	3	**(b)**	-65	÷	5	**(c)**	-18	÷	6	**(d)**	-24	÷	4
	(e)	-48	÷	(-8)	**(f)**	-70	÷	(-10)	**(g)**	-30	÷	(-6)	**(h)**	-72	÷	(-9)

Order of Operations (BODMAS)

BODMAS is the term used to remind us of the order of operations for numerical calculations. The order the letters appear in is the order in which we carry out calculations.

B	Brackets (Calculations inside)	
O	Order (powers and roots)	
D	Division	÷ and × can be swapped order
M	Multiplication	
A	Addition	+ and − can be swapped order
S	Subtraction	

Example 1:
Calculate each of the following:

(a) $8 + 3 \times 5$
$= 8 + 15$
$= 23$

(b) $10 - 8 \div 4$
$= 10 - 2$
$= 8$

(c) 5×3^2
$= 5 \times 9$
$= 45$

Example 2:
Calculate each of the following:

(d) $2(3 + 4)$
$= 2 \times 7$
$= 14$

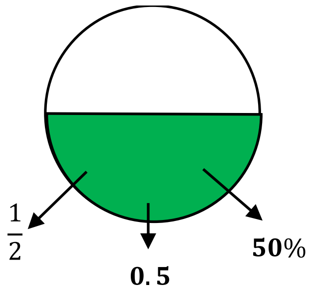
(e) $25 - 3(6 - 4)$
$= 25 - 3 \times 2$
$= 25 - 6$
$= 19$

(f) $1 + 2(3 + 2)^2$
$= 1 + 2 \times 5^2$
$= 1 + 2 \times 25$
$= 1 + 50$
$= 51$

Practice Questions:

1	**(a)** $3 + 4 \times 2$	**(b)** $14 \div 7 + 3$	**(c)** $5 + 2 \times 8$	**(d)** $8 - 15 \div 3$
	(e) $3 + 4^2$	**(f)** $21 - 3^2$	**(g)** $5^2 + 3^2$	**(h)** $4^3 - 2^3$
2	**(a)** 4×2^2	**(b)** 3×4^2	**(c)** $20 \div 2^2$	**(d)** $1 + 2 \times 3^3$
	(e) $2(3 + 1)$	**(f)** $4 + 2(5 - 5)$	**(g)** $9 + 3(6 + 1)$	**(h)** $30 - 5(1 + 2^2)$
3	**(a)** $3 \times (1 + 1)^3$	**(b)** $32 \div (5 - 1)^2$	**(c)** $12 + (5 - 2)^2$	**(d)** $4 + 2(1 + 3)^2$
	(e) $8 \times 4 \div (2 + 2)$	**(f)** $16 \div 2(5 + 3)$	**(g)** $6 + 3(2^3 + 1)$	**(h)** $30 \div 30(2 + 2^2)$

Fractions Decimal Fractions and Percentages



	Benchmarks
Second Level	• Uses knowledge of equivalent forms of common fractions, decimal fractions and percentages, for example, $\frac{3}{4}$ = 0.75 = 75%, to solve problems. • Calculates simple percentages of a quantity, and uses this knowledge to solve problems in everyday contexts, for example, calculates the sale price of an item with a discount of 15%. • Calculates simple fractions of a quantity and uses this knowledge to solve problems, for example, find $\frac{3}{5}$ of 60.
Third Level	• Converts fractions, decimal fractions or percentages into equivalent fractions, decimal fractions or percentages. • Uses knowledge of fractions, decimal fractions and percentages to carry out calculations with and without a calculator. • Solves problems in which related quantities are increased or decreased proportionally. • Expresses quantities as a ratio and where appropriate simplifies, for example, 'if there are 6 teachers and 60 children in a school find the ratio of the number of teachers to the total amount of teachers and children'.
Fourth Level	• Chooses the most efficient form of fractions, decimal fractions or percentages when making calculations. • Uses calculations to support comparisons, decisions and choices. • Calculates the percentage increase or decrease of a value. • Uses knowledge of proportion to solve problems in real-life which involve changes in related quantities.

Fractions

Fractions are used in many areas of Maths and are important in many other areas too; you will see fractions (and percentages) used on a daily basis across media and news streams.

In numeracy we look at simplifying fractions and using the four basic operations with them.

Simplifying

To simplify fractions we first think of sharing: Paul and Jennifer are given £8 between them, Paul gets £4 and Jennifer gets £4. What fraction did each person get?

It's obvious that each person gets $\frac{4}{8}$, this is equivalent to $\frac{1}{2}$. Simplifying is about where this comes from. We create *equivalent fractions* by dividing the top (numerator) and bottom (denominator) by the same value, repeating until the fraction no longer changes.

Example:

Simplify each of the following:

(a) $\frac{5}{15}$

$$= \frac{5}{15} \left(\frac{\div 5}{\div 5}\right)$$
$$= \frac{1}{3}$$

(b) $\frac{24}{36}$

$$= \frac{24}{36} \left(\frac{\div 2}{\div 2}\right)$$
$$= \frac{12}{18} \left(\frac{\div 2}{\div 2}\right)$$
$$= \frac{6}{9} \left(\frac{\div 3}{\div 3}\right)$$
$$= \frac{2}{3}$$

(c) $\frac{16}{40}$

$$= \frac{16}{40} \left(\frac{\div 2}{\div 2}\right)$$
$$= \frac{8}{20} \left(\frac{\div 2}{\div 2}\right)$$
$$= \frac{4}{10} \left(\frac{\div 2}{\div 2}\right)$$
$$= \frac{2}{5}$$

Practice Questions:

Simplify each of the following fractions.

1	**(a)**	$\frac{3}{18}$	**(b)**	$\frac{8}{30}$	**(c)**	$\frac{14}{21}$	**(d)**	$\frac{20}{36}$
	(e)	$\frac{18}{20}$	**(f)**	$\frac{12}{30}$	**(g)**	$\frac{16}{24}$	**(h)**	$\frac{15}{60}$
2	**(a)**	$\frac{25}{100}$	**(b)**	$\frac{48}{60}$	**(c)**	$\frac{26}{143}$	**(d)**	$\frac{85}{300}$
	(e)	$\frac{72}{360}$	**(f)**	$\frac{35}{154}$	**(g)**	$\frac{63}{162}$	**(h)**	$\frac{135}{450}$

Mixed and Improper fractions

Mixed fractions are those that are in common use; stating that a child is $2\frac{1}{2}$ years old for example. However these can be cumbersome in calculations, and so we often use improper fractions (also called top heavy). The $2\frac{1}{2}$ year old child in this case would be $\frac{5}{2}$ years old.

Mixed to Improper:

Take $2\frac{1}{2}$ as our example; this could be worded as 2 and a half cakes. Changing to improper we simply ask, how many cake halves do we have altogether?

We can now see that this is five halves which is $\frac{5}{2}$.

This can be completed more quickly by following the steps below:

- Multiply the whole number by the denominator
- Add the numerator
- Put new number as the numerator

For $2\frac{1}{2}$ this would follow as $(2 \times 2) + 1 = \mathbf{5}$, the new fraction is then $\frac{5}{2}$.

Example 1:

Change each of the following to an improper fraction:

(a) $3\frac{2}{5}$

$3 \times 5 + 2 = 17$

$= \frac{17}{5}$

(b) $4\frac{3}{7}$

$4 \times 7 + 3 = 31$

$= \frac{31}{7}$

(c) $5\frac{5}{8}$

$5 \times 8 + 5 = 45$

$= \frac{45}{8}$

Improper to Mixed:

To change from improper to mixed we simply think about the fraction for what it is, a division.

Take our example from before, if we have $\frac{5}{2}$ cakes, what would this look like as a mixed fraction? This simply says, what is the answer to $5 \div 2$?

Going back to basics we would get 2 remainder 1. In cake terms, we would get 2 cakes with a half left over; therefore $2\frac{1}{2}$.

This can be completed more quickly by following the steps below:

- Divide the numerator by the denominator
- Find the remainder
- Now the result is the whole number and the remainder is the numerator part

Example 1:
Change each of the following to a mixed fraction:

(d) $\frac{8}{3}$
$8 \div 3 = 2\,rem\,2$
$2\frac{2}{3}$

(e) $\frac{19}{4}$
$19 \div 4 = 4\,rem\,3$
$4\frac{3}{4}$

(f) $\frac{39}{7}$
$39 \div 7 = 5\,rem\,4$
$5\frac{4}{7}$

Practice Questions:

1 Change each of the following to a mixed fraction:

(a) $\frac{12}{7}$ **(b)** $\frac{14}{3}$ **(c)** $\frac{15}{2}$ **(d)** $\frac{19}{6}$

(e) $\frac{22}{5}$ **(f)** $\frac{34}{7}$ **(g)** $\frac{67}{4}$ **(h)** $\frac{85}{9}$

2 Change each of the following to an improper fraction:

(a) $1\frac{1}{2}$ **(b)** $3\frac{3}{5}$ **(c)** $4\frac{2}{3}$ **(d)** $2\frac{5}{8}$

(e) $5\frac{6}{7}$ **(f)** $8\frac{3}{7}$ **(g)** $9\frac{4}{5}$ **(h)** $2\frac{14}{15}$

Multiplying

Multiplying Fractions is relatively simple, and works very much as you would expect it to.

To multiply two (or more) fractions, we simply multiply the numerators together and multiply the denominators together.

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Example 1:
Multiply and simplify each of the following:

(a) $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

(b) $\frac{2}{5} \times \frac{3}{10} = \frac{6}{50} \left(\frac{\div 2}{\div 2}\right) = \frac{3}{25}$

(c) $\frac{4}{3} \times \frac{6}{12} = \frac{24}{36} \left(\frac{\div 12}{\div 12}\right) = \frac{2}{3}$

Practice Questions:

1 Multiply and simplify each of the following:

(a) $\frac{1}{4} \times \frac{4}{7}$ **(b)** $\frac{1}{3} \times \frac{3}{10}$ **(c)** $\frac{1}{2} \times \frac{4}{7}$ **(d)** $\frac{2}{3} \times \frac{1}{8}$

(e) $\frac{4}{5} \times \frac{1}{16}$ **(f)** $\frac{6}{7} \times \frac{2}{3}$ **(g)** $\frac{3}{5} \times \frac{10}{21}$ **(h)** $\frac{3}{8} \times \frac{4}{21}$

2 Multiply and simplify each of the following:

(a) $1\frac{1}{4} \times 1\frac{1}{3}$ **(b)** $1\frac{1}{4} \times 1\frac{2}{3}$ **(c)** $2\frac{1}{2} \times 2\frac{1}{2}$ **(d)** $1\frac{3}{4} \times 1\frac{2}{3}$

(e) $3\frac{1}{4} \times 2\frac{1}{5}$ **(f)** $4\frac{1}{3} \times 2\frac{2}{3}$ **(g)** $2\frac{1}{15} \times 2\frac{1}{2}$ **(h)** $3\frac{3}{4} \times 4\frac{1}{5}$

Dividing

Dividing Fractions is relatively simple – using a small trick we can turn each in to a multiply.

To divide two fractions, we simply invert the 2nd fraction (flip it upside down) and calculate as a multiplication.

$$\frac{a}{b} \div \frac{c}{d}$$
$$= \frac{a}{b} \times \frac{d}{c}$$
$$= \frac{a \times d}{b \times c}$$

Example 1:
Divide and simplify each of the following:

(a) $\frac{3}{4} \div \frac{2}{5}$

$\frac{3}{4} \times \frac{5}{2}$

$= \frac{15}{8}$

(b) $\frac{2}{3} \div \frac{7}{6}$

$\frac{2}{3} \times \frac{6}{7}$

$= \frac{12}{21}\left(\frac{\div 3}{\div 3}\right)$

$= \frac{4}{7}$

(c) $\frac{5}{8} \div \frac{4}{9}$

$\frac{5}{8} \times \frac{9}{4}$

$= \frac{45}{32}$

Practice Questions:

1 Multiply and simplify each of the following:

(a)	$\frac{1}{4} \div \frac{4}{7}$	**(b)**	$\frac{1}{3} \div \frac{3}{10}$	**(c)**	$\frac{1}{2} \div \frac{4}{7}$	**(d)**	$\frac{2}{3} \div \frac{1}{8}$
(e)	$\frac{4}{5} \div \frac{1}{16}$	**(f)**	$\frac{6}{7} \div \frac{2}{3}$	**(g)**	$\frac{3}{5} \div \frac{10}{21}$	**(h)**	$\frac{3}{8} \div \frac{4}{21}$

2 Multiply and simplify each of the following:

(a)	$1\frac{1}{4} \div 1\frac{1}{3}$	**(b)**	$1\frac{1}{4} \div 1\frac{2}{3}$	**(c)**	$2\frac{1}{2} \div 2\frac{1}{2}$	**(d)**	$1\frac{3}{4} \div 1\frac{2}{3}$
(e)	$3\frac{1}{4} \div 2\frac{1}{5}$	**(f)**	$4\frac{1}{3} \div 2\frac{2}{3}$	**(g)**	$2\frac{1}{15} \div 2\frac{1}{2}$	**(h)**	$3\frac{3}{4} \div 4\frac{1}{5}$

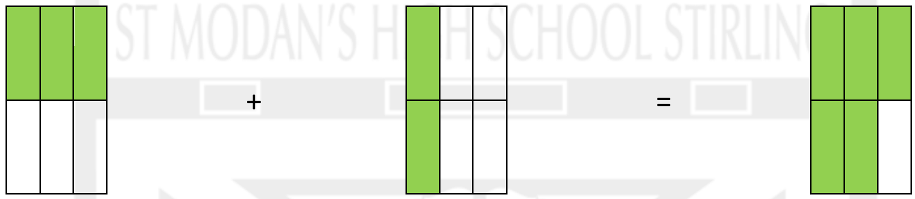
Adding/Subtracting

Adding and subtracting fractions is more complicated than multiplying and dividing. However, if thought of as cakes again we can get a simple understanding of these.

For example $\frac{1}{2}+\frac{1}{3}$ can be modelled with cakes, showing why it is not easy but also showing the solution to it.

You will see that they can't be added because they are of a differing size, it's like adding £ and $ together – we need a common ground.

To find this common ground we try to portion the cake in to comparable parts, as below.



This has created equivalent fractions for each, $\frac{1}{2}=\frac{3}{6}$ and also $\frac{1}{3}=\frac{2}{6}$.
Now we can add these together, $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$

This can be completed more quickly by following the steps below:

- Find a number which both denominators fit perfectly in to
- Use this number as the denominator of your equivalent fractions
- Add or subtract as normal

Example 1:
Complete the following calculations, leave your answer as simplified fractions:

(d) $\frac{1}{4}+\frac{2}{5}$

Both 4 and 5 fit into 20

$=\frac{5}{20}+\frac{8}{20}$

$=\frac{13}{20}$

(e) $\frac{2}{3}-\frac{2}{5}$

Both 3 and 5 fit into 15

$=\frac{10}{15}-\frac{6}{15}$

$=\frac{4}{15}$

(f) $\frac{1}{2}+\frac{2}{3}-\frac{1}{5}$

2, 3 and 5 all fit into 30

$=\frac{15}{30}+\frac{20}{30}-\frac{6}{30}$

$=\frac{29}{30}$

Practice Questions:

1 Complete the following calculations, leave your answer as simplified fractions:

(a) $\frac{1}{4}+\frac{4}{7}$ **(b)** $\frac{1}{3}+\frac{3}{10}$ **(c)** $\frac{1}{2}+\frac{4}{7}$ **(d)** $\frac{2}{3}+\frac{1}{8}$

(e) $\frac{4}{5}-\frac{1}{16}$ **(f)** $\frac{6}{7}-\frac{2}{3}$ **(g)** $\frac{3}{5}-\frac{10}{21}$ **(h)** $\frac{3}{8}-\frac{4}{21}$

2 Complete the following calculations, leave your answer as simplified fractions:

(a) $1\frac{1}{4}+1\frac{1}{3}$ **(b)** $1\frac{1}{4}+1\frac{2}{3}$ **(c)** $2\frac{1}{2}+2\frac{1}{2}$ **(d)** $1\frac{3}{4}+1\frac{2}{3}$

(e) $3\frac{1}{4}-2\frac{1}{5}$ **(f)** $4\frac{1}{3}-2\frac{2}{3}$ **(g)** $2\frac{1}{2}-2\frac{1}{15}$ **(h)** $4\frac{1}{5}-3\frac{3}{4}$

Percentages

Percentages are used and visible in many areas of daily life, from the daily news to the discounts in shops. It is important that pupils can understand what a percentage is and how to calculate percentages of quantities.

To calculate a percentage we can use many different models; once we are comfortable with these we can decide which is best for a particular case/situation.

It is useful to recognise the link between Fractions, Decimals and Percentages. Percentages can be written as fractions of 100; from earlier, you will recall that fractions can be simplified and they are actually divisions. Some simple conversions are shown below.

Percentage	Fraction	Decimal
50%	$\frac{50}{100} = \frac{1}{2}$	$1 \div 2 = 0.5$
25%	$\frac{25}{100} = \frac{1}{4}$	$1 \div 4 = 0.25$
75%	$\frac{75}{100} = \frac{3}{4}$	$3 \div 4 = 0.75$
10%	$\frac{10}{100} = \frac{1}{10}$	$1 \div 10 = 0.1$
20%	$\frac{20}{100} = \frac{1}{5}$	$1 \div 5 = 0.2$
5%	$\frac{5}{100} = \frac{1}{20}$	$1 \div 20 = 0.05$
1%	$\frac{1}{100}$	$1 \div 100 = 0.01$

Example 1 (partitioning):
Calculate 35% of £180.

$$35\% = 30\% + 5\%$$
$$30\% = 3 \times 10\%$$
$$5\% = 4 \times 1\%$$

10% is simply $100\% \div 10$
1% is simply $100\% \div 100$

In our case, $10\% = £180 \div 10 = £18$

$1\% = £180 \div 100 = £1.80$

$30\% = 3 \times £18 = £54$

$5\% = 5 \times £1.80 = £9$

$$34\% = £54 + £9 = £63$$

Example 2 (fractions):
Calculate 35% of £180.

$$35\% \; of \; £180$$
$$= \frac{35}{100} \; of \; £180$$
$$= \frac{7}{20} \; of \; £180$$

$$\frac{1}{20} \; of \; £180 = £9$$
$$\therefore \frac{7}{20} \; of \; £180 = £63$$

Example 3 (decimals):
Calculate 35% of £180.

$$0.35 \times £180$$

	30	**5**
100	3000	500
80	2400	400

$3000 + 2400 = 5400$

$500 + 400 = 900$

$5400 + 900 = 6300$

Bringing back the decimal point we get £63.00

Any of the methods above can be used to find the answer to the following practice questions; in some cases one method will be easier than the others.

Practice Questions:

Select the method you feel is best in each case to calculate the following and work out the answer:

1	**(a)**	$25\% \ of \ £200$	**(b)**	$10\% \ of \ £320$	**(c)**	$40\% \ of \ 450kg$	**(d)**	$60\% \ of \ 380g$
	(e)	$15\% \ of \ \$60$	**(f)**	$30\% \ of \ £50$	**(g)**	$90\% \ of \ €150$	**(h)**	$35\% \ of \ 180kg$
2	**(a)**	$32\% \ of \ 550g$	**(b)**	$46\% \ of \ £420$	**(c)**	$79\% \ of \ 400kg$	**(d)**	$83\% \ of \ 150g$
	(e)	$7\% \ of \ \$40$	**(f)**	$52\% \ of \ 460g$	**(g)**	$105\% \ of \ €350$	**(h)**	$124\% \ of \ 600ml$

Money

	Benchmarks
Second Level	• Carries out money calculations involving the four operations. • Compares costs and determines affordability within a given budget. • Demonstrates understanding of the benefits and risks of using bank cards and digital technologies. • Calculates profit and loss accurately, for example, when working with a budget for an enterprise activity.
Third Level	• Demonstrates understanding of best value in relation to contracts and services when comparing products. • Chooses the best value for their personal situation and justifies choices. • Budgets effectively, using digital technology where appropriate, showing development of financial capability. • Demonstrates knowledge of financial terms, for example, debit/credit, APR, pa, direct debit/standing order and interest rate. • Converts between different currencies.
Fourth Level	• Applies understanding of credit and debit in relation to earnings and deductions. • Uses budgeting skills to manage income effectively and justifies spending and saving choices. • Calculates net income by selecting appropriate information. • Compares a range of personal finance products. • Communicates the impact of financial decisions. • Applies knowledge of currency conversion to determine best value.

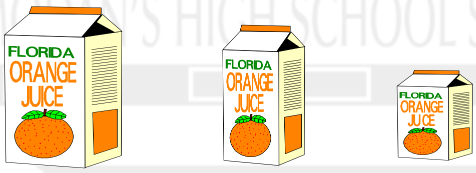
Money

Working with money is an area that pupils understand well, and are particularly interested in!

In the main, the techniques and calculations as set out previously (number processes, fractions, decimals and percentages) are the skills that are applied when working with money. In this section we will focus on two main areas; calculating the *best buy* when given a range of choices and looking at *foreign exchange*.

Best Buy
When buying products it makes sense to check that you are getting value for money, we usually refer to this as the best buy.

Example 1:
The volume and prices for 3 different cartons of orange juice are shown below. Which one provides best value for money? Justify your answer.



3 litres £1.74 | 1·5 litres 84p | 0·8 litres 48p

$$3 \; litres = £1.74$$
$$1 \; litre = £1.74 \div 3$$
$$1 \; litre = £0.58$$

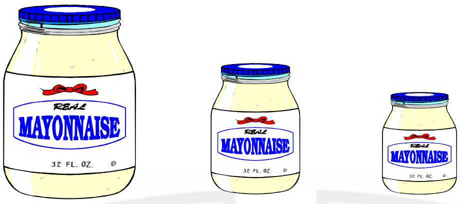
$$1.5 \; litres = £0.84$$
$$1 \; litre = £0.84 \div 1.5$$
$$1 \; litre = £0.56$$

$$0.8 \; litres = £0.48$$
$$1 \; litre = £0.48 \div 0.8$$
$$1 \; litre = £0.60$$

We can now see that for 1 litre of orange juice we could pay 58p, 56p or 60p.

Therefore the 1.5 litre carton provides the best value for money as it costs 56p per litre compared to 58p and 60p per litre for the other carton sizes.

Example 2:
The weight and prices for 3 different jars of mayonnaise are shown below. Which one provides best value for money? Justify your answer.



600 g	320 g	200 g
£2.50	£1.40	£1.05

$$£2.50 = 600g$$
$$£1 = 600g \div 2.5$$
$$£1 = 240g$$

$$£1.40 = 320g$$
$$£1 = 320g \div 1.4$$
$$£1 = 228.6g$$

$$£1.05 = 200g$$
$$£1 = 200g \div 1.05$$
$$£1 = 190.5g$$

We can now see that for £1 we could get 240g, 228.6g or 190.5g.

Therefore the 600g jar provides the best value for money as it gives 240g per £1 compared to 228.6g and 190.5g per £1 for the other jar sizes.

Practice Questions:

Find the best value for money item in each case below:

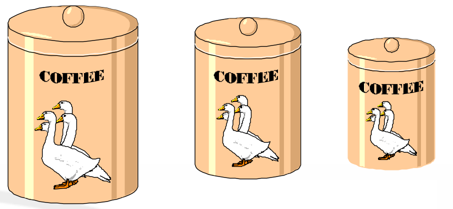
(a)

3 litres
£1.68

2 litres
£1.20

1 litre
£0.62

(b)



550 g
£8.20

350 g
£5.60

200 g
£3.60

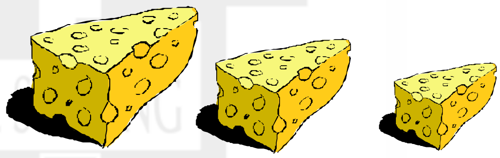
(c)

$0 \cdot 7$ litres
40p

2 litres
£1.39

4 litres
£2.56

(d)

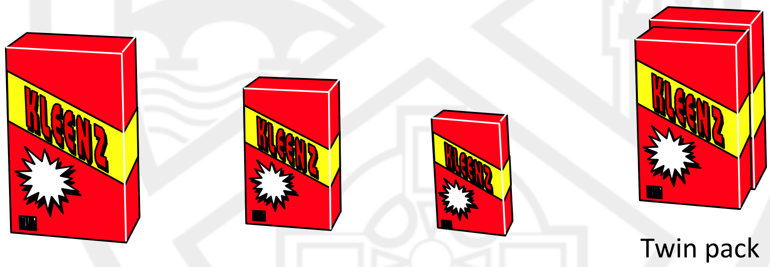


$0 \cdot 8\ kg$
£4.20

$0 \cdot 5\ kg$
£2.80

$0 \cdot 3\ kg$
£1.59

(e)



6 kg
£10.50

$2 \cdot 5\ kg$
£4.80

$1 \cdot 5\ kg$
£3.06

Twin pack
$(2 \times 3 \cdot 5\ kg)$
£11.90

Foreign Exchange

We use foreign exchange to convert money between currencies. The *exchange rate* is used to multiply or to divide to make this possible.

To go from £ to another currency, we multiply by the exchange rate.
To go from another currency to £ we divide by the exchange rate.

Example 1:
The McLaren family are travelling to Spain for a holiday. They want to take £600 spending money with them. Given that the exchange rate is
£1 = €1.12, calculate how many Euros this will get them.

600×1.12
$= 672$

They would get €672 for their £600.

Example 2:
Ellie has $230 left over from her holiday in Florida. Given that the exchange rate is £1 = $1.31, calculate how many pounds she will get back.

$230 \div 1.31$
$= 175.57$

She would get £175.57 for her $230.

Practice Questions:

The exchange rates for £ to a selection of other currencies are shown below:

XE Live Exchange Rates

53

United Kingdom

Inverse	EUR	USD	AUD	CAD	INR	AED	ZAR
1 GBP	1.13062	1.32636	1.75954	1.70007	86.2449	4.87155	18.7118

1 **(a)** Calculate how many Euros we would get for £250.

(b) Calculate how many South African Rand we would get for £480.

(c) How many more Australian Dollars would we get for £370 than Canadian Dollars?

2 **(a)** Calculate how many Pounds we would get for 720 US Dollars.

(b) Calculate how many Pounds we would get for 1500 UAE Dirham.

(c) How many less Pounds would we get for 400 Canadian Dollars than 400 Australian Dollars?

Time

	Benchmarks
Second Level	• Reads and records time in both 12 hour and 24 hour notation and converts between the two. • Knows the relationships between commonly used units of time and carries out simple conversion calculations, for example, changes $1\frac{3}{4}$ hours into minutes. • Uses and interprets a range of electronic and paper-based timetables and calendars to plan events or activities and solve real life problems. • Calculates durations of activities and events including situations bridging across several hours and parts of hours using both 12 hour clock and 24 hour notation. • Estimates the duration of a journey based on knowledge of the link between speed, distance and time. • Chooses the most appropriate timing device in practical situations and records using relevant units, including hundredths of a second. • Selects the most appropriate unit of time for a given task and justifies choice.
Third Level	• Applies knowledge of the relationship between speed, distance and time to find each of the three variables. • Calculates time durations across hours and days.
Fourth Level	• Demonstrates effective time management skills, for example, working with different time zones or making plans, including across midnight. • Carries out calculations involving speed, distance and time involving decimal fraction hours. • Calculates time durations across hours, days and months.

Time Differences

It is important for everyday life that we can accurately calculate time differences;

- How long is it until?
- How long ago was that?

  etc

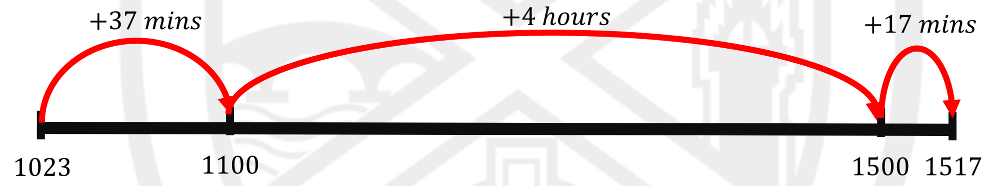
For this we use the technique of the empty number line again (used in addition and subtraction). We also need to know some basic facts.

1 minute	=	60 seconds
1 hour	=	60 minutes
1 day	=	24 hours
1 week	=	7 days
1 year	=	52 weeks
1 year	=	365 days
1 leap year	=	366 days

We also need to know the number of days in each month – there are various ways of remembering this but the rhyme below is the best known way of doing so.

Thirty days has September,
April, June and November.
All the rest have thirty one,
Except for February alone;
which had twenty days clear
and 29 in a leap year.

Example 1:
How long is it from 1023 to 1517



As you will see, we have added 4 hours and 54 minutes.

4h and 54m

Example 2:
Today is October 18th, how many days are there until Christmas?

Days left in October = 13 days
November = 30 days
December (to Christmas) = 25 days

Therefore there are 68 days until Christmas.

Practice Questions:

1 Calculate how long it is from:

(a)	$2:35pm\ to\ 3:05pm$	**(b)**	$0628\ to\ 0737$	**(c)**	$9:26am\ to\ 10:43am$
(d)	0927 to 1115	**(e)**	9:25am to 1:14pm	**(f)**	2346 to 0617
(g)	1^{st} January to 24^{th} February	**(h)**	3^{rd} May to 18^{th} July	**(i)**	28^{th} June to Bonfire Night

Fractional and Decimal Time

In calculations it can be beneficial to have time written as fractions of hours or as decimal hours, we can convert to and from these in the same way as we did with percentages. The only difference being that time relates to hours (the number 60) as opposed to the 100 we used with percentages.

Length of Time	Fraction	Decimal
30 minutes	$\frac{30}{60} = \frac{1}{2}$ *of an hour*	$1 \div 2 = 0.5$ *of an hour*
15 minutes	$\frac{15}{60} = \frac{1}{4}$ *of an hour*	$1 \div 4 = 0.25$ *of an hour*
45 minutes	$\frac{45}{60} = \frac{3}{4}$ *of an hour*	$3 \div 4 = 0.75$ *of an hour*
36 minutes	$\frac{36}{60} = \frac{3}{5}$ *of an hour*	$3 \div 5 = 0.6$ *of an hour*
23 minutes	$\frac{23}{60} = \frac{23}{60}$ *of an hour*	$23 \div 60 = 0.383$ *of an hour*
1h 54 minutes	$\frac{114}{60} = \frac{19}{10}$ *of an hour*	$19 \div 10 = 1.9$ *of an hour*
3 h 6 minutes	$\frac{186}{60} = \frac{31}{10}$ *of an hour*	$31 \div 10 = 3.1$ *of an hour*

Practice Questions:

1 Express each of the following in hours only, rounding your answers to 2 decimal places where necessary:

(a) 36 minutes	**(b)** 57 minutes	**(c)** 20 minutes
(d) 1 hour 18 minutes	**(e)** 2 hours 6 minutes	**(f)** 5 hours 48 minutes
(g) 2 hours 21 minutes	**(h)** 9 hours 55 minutes	**(i)** 22 hours 8 minutes

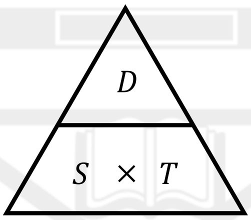
Speed Distance and Time

Speed, Distance and Time are directly linked; a car that travels 30 miles in an hour must have done so at an average speed of 30mph. This basic fact allows us to create formulae (calculation methods) to find a missing Speed, Distance or Time.

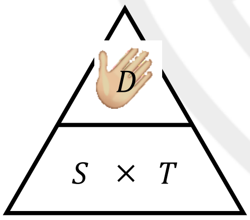
The three formulae we use are:

$Distance = Speed \times Time$	$D = S \times T$
$Speed = \dfrac{Distance}{Time}$	$S = \dfrac{D}{T}$
$Time = \dfrac{Distance}{Speed}$	$T = \dfrac{D}{S}$

We usually use a triangle to help us remember these:

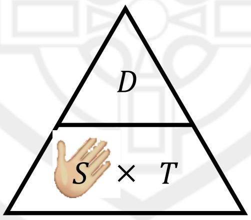


When asked to find Speed, Distance or Time, we cover it up the one we are looking for and the calculation this leaves is the one we use:



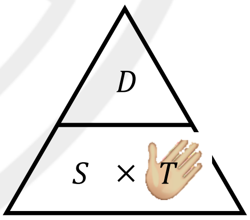
Indicates:

$D = S \times T$



Indicates:

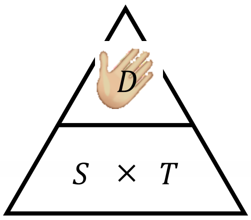
$S = \dfrac{D}{T}$



Indicates:

$T = \dfrac{D}{S}$

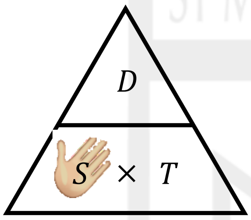
Example 1:
A train travels at an average speed of 80mph for 1h 30 minutes, how far has it travelled?



We need the time in hours to make it consistent with the units given. This would be 1.5 hours (prior skill).

$$D = S \times T$$
$$D = 80 \times 1.5$$
$$D = 120\ miles$$

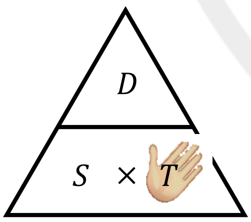
Example 2:
A businessman drives from Stirling to London, a journey of 660km, in a time of 8h and 15 minutes. He takes a few breaks along the way, calculate his average seed for the entire journey.



We need the time in hours to make it consistent with the units required (kmph makes most sense). This would be 8.25 hours (prior skill).

$$S = \frac{D}{T}$$
$$S = \frac{660}{8.25}$$
$$S = 80kmph$$

Example 3:
How long would a journey of 160miles take at an average speed of 50mph?



$$T = \frac{D}{S}$$
$$S = \frac{160}{50}$$
$$S = 3.2\ hours$$

We need to convert 3.2 hours in to a more sensible unit. Reversing the process from before, this would be 3h 12minutes.

Practice Questions:

1. Calculate the distance travelled for each journey below.
How far have you gone if you travel for…

(a) 4 hours at a speed of 50 km/h?

(b) 6 hours at a speed of 65 mph?

(c) $2\frac{1}{2}$ hours at a speed of 87 km/h?

(d) 40 minutes at a speed of 300 metres per minute? (answer in kilometres)

2. **(a)** A car travels a distance of 340 km in 4 hours 36 minutes.
Calculate its average speed.

(b) A plane travels a distance of 490 km in 1 hours 15 minutes.
Calculate its average speed.

(c) A car travels a distance of 58 miles in 48 minutes.
Calculate its average speed.

(d) A boat travels a distance of 86 km in 9 hours 6 minutes.
Calculate its average speed.

(e) A man runs a distance of 60 km in 5 hours 17 minutes.
Calculate his average speed.

3. **(a)** A car travels a distance of 340 km at an average speed of 64 km/h.
Calculate the time taken for the journey, giving your answer to the nearest minute.

(b) A plane travels a distance of 3123 km at an average speed of 278 km/h.
Calculate the time taken for the journey, giving your answer to the nearest minute.

(c) A car travels a distance of 58 miles at an average speed of 48 km/h.
Calculate the time taken for the journey, giving your answer to the nearest minute.

(d) A boat travels a distance of 186 km at an average speed of 18 km/h.
Calculate the time taken for the journey, giving your answer to the nearest minute.

(e) A man runs a distance of 32 km at an average speed of 20 km/h.
Calculate the time taken for the journey, giving your answer to the nearest minute.

Measurement

	Benchmarks
Second Level	• Uses the comparative size of familiar objects to make reasonable estimations of length, mass, area and capacity. • Estimates to the nearest appropriate unit, then measures accurately: length, height and distance in millimetres (mm), centimetres (cm), metres (m) and kilometres (km); mass in grams (g) and kilograms (kg); and capacity in millilitres (ml) and litres (l). • Calculates the perimeter of simple straight sided 2D shapes in millimetres (mm), centimetres (cm) and metres (m). • Calculates the area of squares, rectangles and right-angled triangles in square millimetres (mm^2), square centimetres (cm^2) and square metres (m^2). • Calculates the volume of cubes and cuboids in cubic centimetres (cm^3) and cubic metres (m^3). • Converts between common units of measurement using decimal notation, for example, 550 cm = 5·5 m; 3·009 kg = 3009 g. • Chooses the most appropriate measuring device for a given task and carries out the required calculation, recording results in the correct unit. • Reads a variety of scales accurately. • Draws squares and rectangles accurately with a given perimeter or area. • Demonstrates understanding of the conservation of measurement, for example, draw three different rectangles each with an area of 24 cm^2. • Shows awareness of imperial units used in everyday life, for example, miles or stones.

Third Level	• Chooses appropriate units for length, area and volume when solving practical problems. • Converts between standard units to three decimal places and applies this when solving calculations of length, capacity, volume and area.

Fourth Level	• Demonstrates understanding of the impact of truncation and premature rounding.

Measurement

Understanding measure, and being able to convert between different measurements, is a useful skill for life and for work.

The units used for measuring length, weight, area and volume are defined by the metric system, and there are a few exceptions in everyday life where this is not used. However, here we will focus on the standard metric units only.

We will look at measurement units, conversions and basic calculations where measure is important.

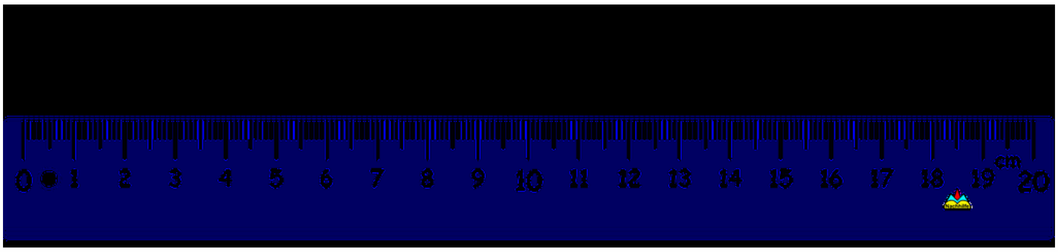
Length and Weight
Standard Conversions:

Length	1 centimetre (1cm)	=	10 millimetres (10mm)
	1 metre (1m)	=	100 centimetres (100cm)
	1 kilometre (1km)	=	1000 metres (1000m)
Weight	1 gram (1g)	=	1000 milligrams (1000mg)
	1 kilogram (1kg)	=	1000 grams (1000g)
	1 Tonne (1T)	=	1000 kilograms (1000kg)

Example 1:
Convert the following:

(a) 38mm to cm	(b) 2400g to kg	(c) 215cm to m
Centimetres are *larger* than millimetres so we would have less of them… divide! $38 \div 10$ $= 3.8cm$	Kilograms are *larger* than grams so we would have less of them… divide! $2400 \div 1000$ $= 2.4\text{kg}$	Metres are *larger* than centimetres so we would have less of them… divide! $215 \div 100$ $= 2.15cm$
(d) 24cm to mm	**(e) 2.9km to m**	**(f) 8.6T to kg**
Millimetres are *smaller* than centimetres so we would have more of them… multiply! 24×10 $= 240\text{mm}$	Metres are *smaller* than kilometres so we would have more of them… multiply 2.9×1000 $= 2900\text{m}$	Kilograms are *smaller* than Tonnes so we would have more of them… multiply 8.6×1000 $= 8600\text{kg}$

Example 2:
What length does each arrow point to in mm?



$$a = 5.1\text{cm}, b = 9.6\text{cm and } d = 16.7\text{cm}$$

What would these values be in millimetres?

$$a = 51\text{mm}, b = 96\text{mm and } d = 167\text{mm}$$

Area and Volume
Standard Conversions:

1 cubic centimetre (1cm^3)	=	1 millilitre (1ml)
1000 cubic centimetres (1000cm^3)	=	1 litre (1l)
1000 millilitre (1000ml)	=	1 litre (1l)

These conversions are less known and less used, but are nice to know nonetheless!

Area	1 square centimetre (1cm^2)	=	100 square millimetres (100mm^2)
	1 square metre (1m^2)	=	10 000 square centimetres(10cm^2)
	1 square kilometre (1km^2)	=	1 million square metres ($1\,000\,000\text{m}^2$)
Volume	1 cubic centimetre (1cm^3)	=	1 000 cubic millimetres (1000mm^3)
	1 cubic metre (1m^3)	=	1 million cubic centimetres(10cm^3)
	1 cubic kilometre (1km^3)	=	1 billion cubic metres ($1\,000\,000\,000\text{m}^3$)

Example 1:
Convert the following:

(a) 2.6l to ml	(b) 680ml to l	(c) $6250\text{cm}^3\text{ to l}$
Millilitres are *smaller* than litres so we would have more of them… multiply!	Litres are *larger* than millilitres so we would have less of them… divide!	Litres are *larger* than cubic centimetres so we would have less of them… divide!
2.6×1000 $= 2600\text{ml}$	$680 \div 1000$ $= 0.68\text{l}$	$6250 \div 1000$ $= 6.25\text{l}$

Practice Questions:

Convert each of the following:

1	**(a)**	$25cm\ to\ mm$	**(b)**	$2.3m\ to\ cm$	**(c)**	$4.8km\ to\ m$	**(d)**	$23mm\ to\ cm$
	(e)	$470m\ to\ km$	**(f)**	$305cm\ to\ m$	**(g)**	$30mm\ to\ m$	**(h)**	$240cm\ to\ km$
2	**(a)**	$2kg\ to\ g$	**(b)**	$3.4T\ to\ kg$	**(c)**	$400g\ to\ kg$	**(d)**	$600mg\ to\ g$
	(e)	$230kg\ to\ T$	**(f)**	$35g\ to\ mg$	**(g)**	$460kg\ to\ T$	**(h)**	$5600g\ to\ T$
3	**(a)**	$24cm^3\ to\ ml$	**(b)**	$2.3l\ to\ ml$	**(c)**	$2500cm^3\ to\ l$	**(d)**	$5l\ to\ cm^3$
	(e)	$6000ml\ to\ l$	**(f)**	$3.8l\ to\ cm^3$	**(g)**	$25ml\ to\ l$	**(h)**	$95cm^3 to\ l$

Area and Volume Calculations

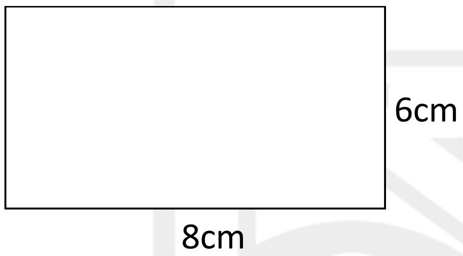
Areas and volumes of most basic shapes can be found using a series of small formulae. These are as follows:

Area of a Rectangle:	$\boldsymbol{A = lb}$	Where l is the length and b is the breadth.
Area of a Triangle:	$\boldsymbol{A = \frac{1}{2}lb}$	Where l is the length and b is the breadth.
Area of a Circle:	$\boldsymbol{A = \pi r^2}$	Where π is 3.14 (or just π on a calculator) and r is the radius.
Volume of a prism	$\boldsymbol{V = Ah}$	Where A is the area of the surface and h is the height.

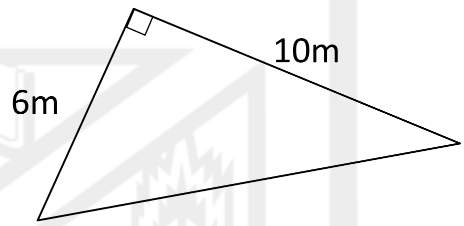
Most shapes constructed of straight lines can be broken up in to rectangles and triangles.

Example 1:

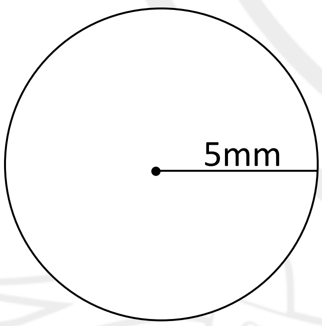
Calculate the area of each of the following shapes.



$$A = lb$$
$$A = 8 \times 6$$
$$A = 48cm^2$$

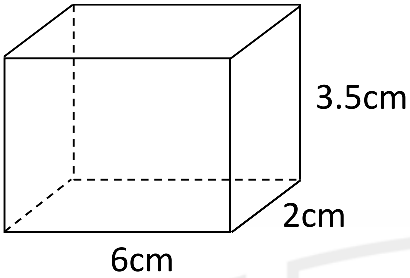


$$A = \frac{1}{2}lb$$
$$A = \frac{1}{2} \times 6 \times 10$$
$$A = 30m^2$$



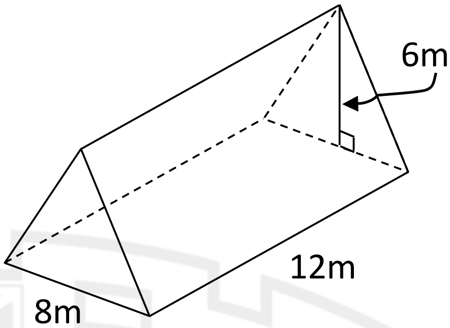
$$A = \pi r^2$$
$$A = 3.14 \times 5^2$$
$$A = 3.14 \times 25$$
$$A = 78.5mm^2$$

Example 2:
Calculate the volume of each of the following shapes.



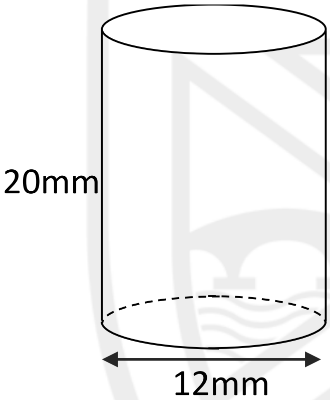
$$A = lb$$
$$A = 6 \times 2$$
$$A = 12cm^2$$

$$V = Ah$$
$$V = 12 \times 3.5$$
$$V = 42cm^3$$



$$A = \frac{1}{2}lb$$
$$A = \frac{1}{2} \times 8 \times 6$$
$$A = 24m^2$$

$$V = Ah$$
$$V = 24 \times 12$$
$$V = 288m^3$$

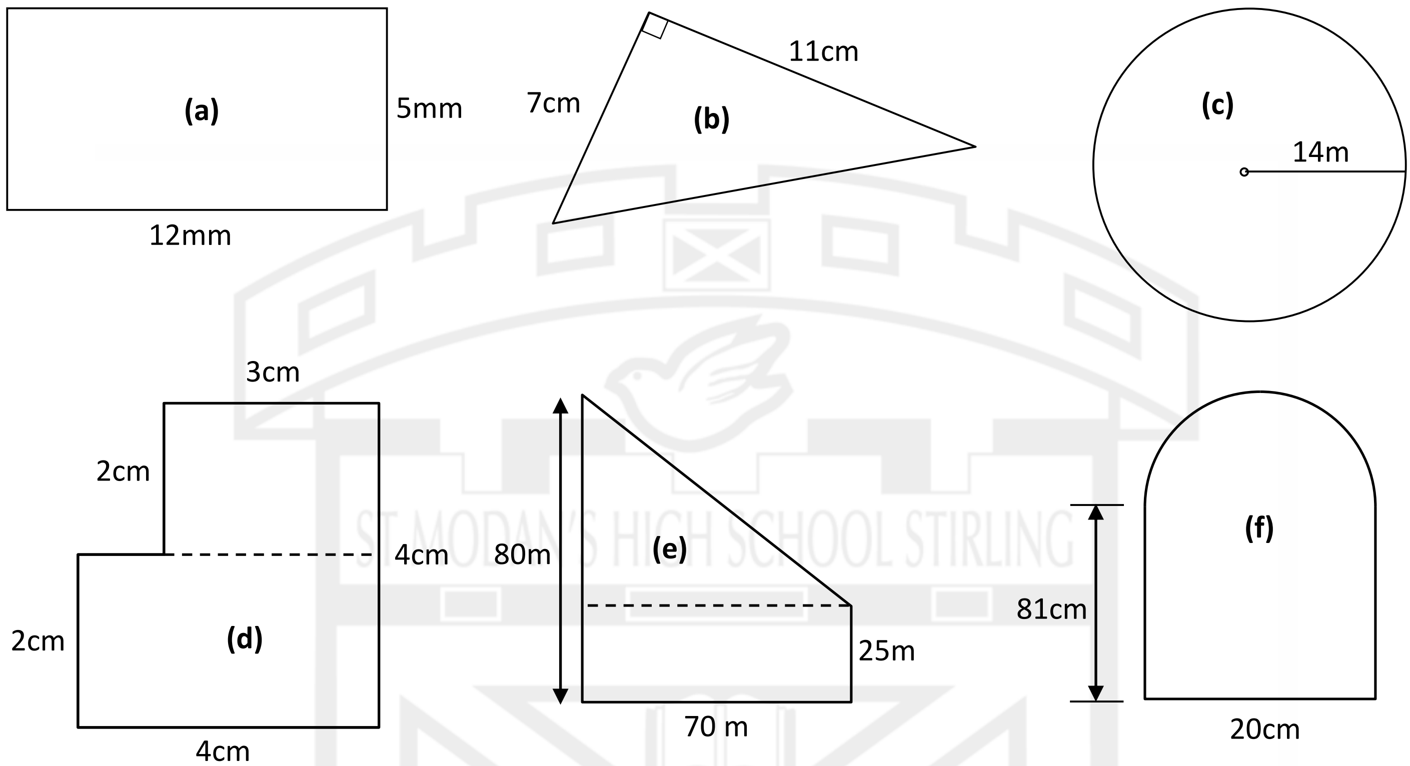


$$A = \pi r^2$$
$$A = 3.14 \times 6^2$$
$$A = 3.14 \times 36$$
$$A = 113.04mm^2$$

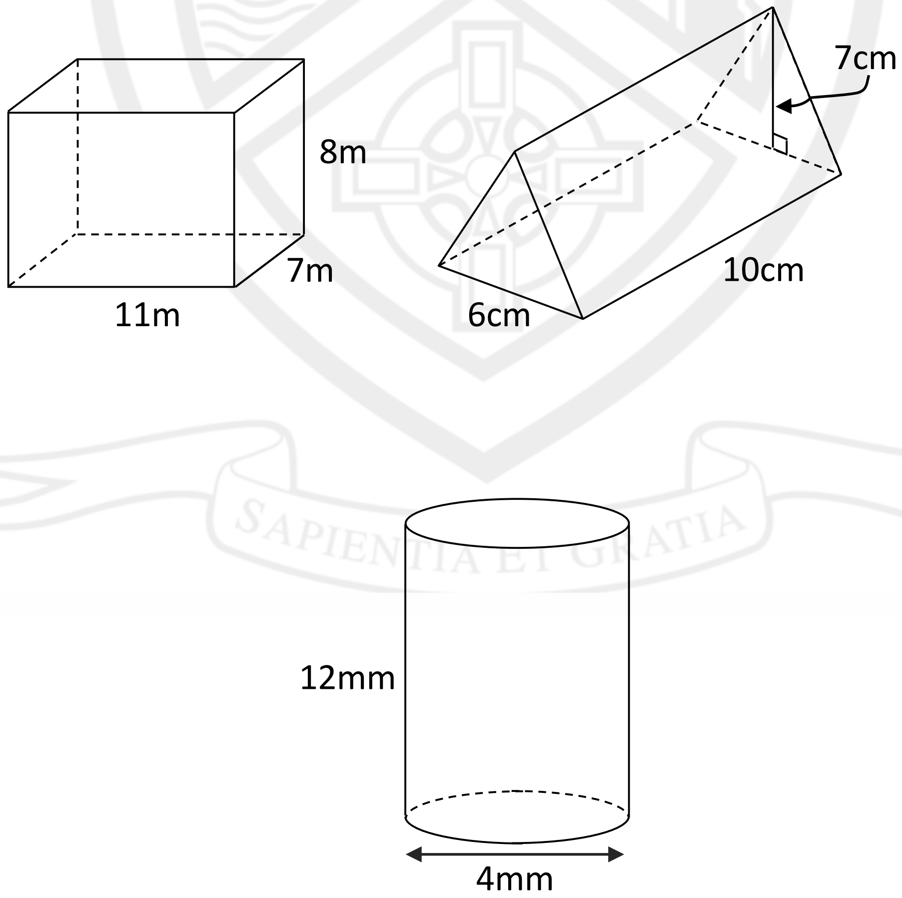
$$V = Ah$$
$$V = 113.04 \times 20$$
$$V = 2260.8mm^3$$

Practice Questions

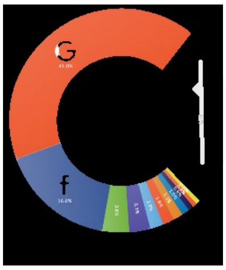
1. Calculate the area of each of the following shapes:



2. Calculate the volume of each of the following shapes:



Data and Analysis



	Benchmarks
Second Level	• Devises ways of collecting data in the most suitable way for the given task. • Collects, organises and displays data accurately in a variety of ways including through the use of digital technologies, for example, creating surveys, tables, bar graphs, line graphs, frequency tables, simple pie charts and spreadsheets. • Analyses, interprets and draws conclusions from a variety of data. • Draws conclusions about the reliability of data taking into account, for example, the author, the audience, the scale and sample size used.
Third Level	• Sources information or collects data making use of digital technology where appropriate. • Interprets data sourced or given. • Describes trends in data using appropriate language, for example, increasing trend. • Determines if information is robust, vague or misleading by considering, for example, the validity of the source, scale used, sample size, method of presentation and appropriateness of how the sample was selected.
Fourth Level	• Interprets raw and graphical data. • Uses statistical language, for example, correlations, to describe identified relationships.

Averages

There are 3 forms of averages in common usage; the mean, the median and the mode. At times data is also described using its spread, this is called the range.

Mean	The sum of all data divided by the number of pieces of data.
Median	The middle number in an ordered list.
Mode	The most commonly occurring piece of data.
Range	The highest number subtract the smallest number in a data set.

Example 1:
The price of a pint of milk in 5 different shops is found to be 50p, 45p, 54p, 48p and 55p.

Calculate the man price of a mint of milk in these shops.

$$Mean = \frac{Sum\ of\ all\ data}{Number\ of\ pieces\ of\ data}$$
$$Mean = \frac{50 + 45 + 54 + 48 + 55}{5}$$
$$Mean = \frac{252}{5}$$
$$Mean = 50.4p$$

Example 2:
Calculate the median of the numbers: 1, 3, 4, 4, 4, 5, 5, 7, 8, 12, 14.

The middle number in this list can be found by scoring out sets of numbers from each end until you leave a single number in the middle:

~~1~~, 3, 4, 4, 4, 5, 5, 7, 8, 12, ~~14~~
becomes
~~1, 3, 4, 4, 4~~, 5, ~~5, 7, 8, 12, 14~~

Leaving the median of 5, the middle number.

Example 3:
Calculate the median of the numbers: 1, 3, 3, 4, 4, 4, 5, 5, 7, 8, 12, 14.

The middle number in this list can be found by scoring out sets of numbers from each end until you leave a single number in the middle:

~~1,~~ 3, 3, 4, 4, 4, 5, 5, 7, 8, 12, ~~14~~
becomes
~~1, 3, 3, 4, 4~~, 4, 5, ~~5, 7, 8, 12, 14~~

This time leaving two numbers in the middle; 4 and 5.
The median is halfway between these, therefore 4.5 in this case.

Example 4:
Find the mode of the following list of favourite TV colours:

Purple	Black	Purple	Pink	Purple
Purple	Pink	Black	Black	Red
Green	Black	Pink	Green	Red
Black	Purple	Red	Green	Red
Green	Green	Pink	Green	Purple
Green	Red	Pink	Red	Pink

A simple frequency table can help us find the mode:

Colour	Tally	Frequency
Red	𝍸 I	6
Green	𝍸 II	7
Black	𝍸	5
Pink	𝍸 I	6
Purple	𝍸 I	6

We can now see clearly that the most common colour, the mode, is Green.

Example 5:
Calculate the range of the numbers: 1, 3, 4, 4, 4, 5, 5, 7, 8, 12, 14.

The highest number is 14 and the smallest number is 1.
Therefore the range is 13.

Practice Questions

1. Find the mean, median, mode and range for each of the following data sets. (Remember to write the numbers in order before finding the median)

(a)	7	6	3	11	8	7	10	4	7					
(b)	1	3	11	4	9	15	7	2	6	3	5			
(c)	2.0	2.5	3.3	1.7	2.2	2.7	1.9	2.2	2.9	1.5	2.4			
(d)	85	81	80	89	88	81	85	86	81	90				
(e)	4	2	3	1	2	4	3	2	1	2	2	3	2	4
(f)	1.2	0.8	2.0	0.9	0.8	0.6	1.1	2.2	1.2	0.8	0.9	1.9		
(g)	332	308	340	325	336	341	319	324	317	306	308	320		
(h)	8.8	12.4	15.2	10.3	11.9	9.7	20.0	16.9	9.7	17.1				

2. The frequency table shows the results of a survey conducted in a block of flats to find out how many people were living in each house.

number of people in flat	frequency
1	3
2	5
3	12
4	3
5	1
Total	24

(a) Use the table to calculate the Median and Range.

(b) What is the modal number of people living in a flat?

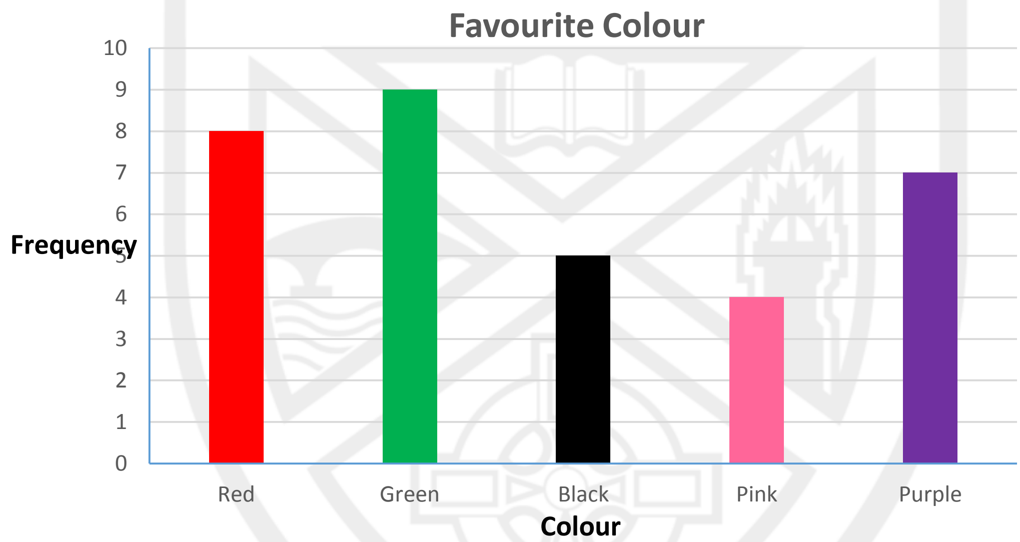
Graphical Displays

It is common place to see data displayed in graphs and charts; news outlets use these to daily and any visit to social media sites/outlets will expose users to many. For this reason it is important that we all have the ability to understand these graphs/charts and recognise where they may be misleading.

Here we will look at 4 common graphical displays; bar charts, line graphs, pie charts and scatter graphs.

Example 1 (bar chart):
Construct a bar chart using the data of favourite colours (similar to the data we used before)

Colour	Tally	Frequency
Red	𝍸 |||	8
Green	𝍸 ||||	9
Black	𝍸	5
Pink	||||	4
Purple	𝍸 ||	7

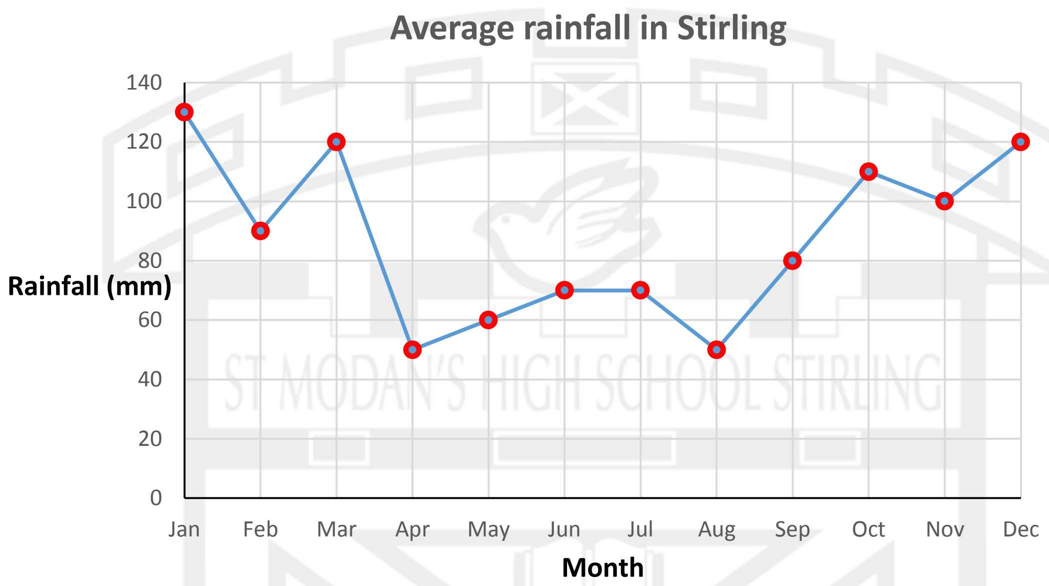


Notice that the bars are of equal width and that there are gaps at the start, the end and between the bars.

Example 2 (line graph):
The rainfall in millimetres were recorded for Stirling over the period of a year. Show this information on a line graph.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
130	90	120	50	60	70	70	60	80	110	100	120

Firstly we plot points, then join these up to create a line.



Example 3 (pie chart):
A group of pupils were surveyed to find out about their favourite social media apps; the table below shows the results.

Media	Frequency
Twitter	5
Instagram	9
Snapchat	16
Facebook	10

We need to find the angle each app would occupy on a pie chart.

- Find the total amount of data
- Find each app as a fraction of this
- The angle will be this fraction of a full circle $(360°)$

Media	Frequency	Angle
Twitter	5	$\frac{5}{40}\ of\ 360° = 45°$
Instagram	9	$\frac{9}{40}\ of\ 360° = 81°$
Snapchat	16	$\frac{16}{40}\ of\ 360° = 144°$
Facebook	10	$\frac{10}{40}\ of\ 360° = 90°$
Totals	40	360°

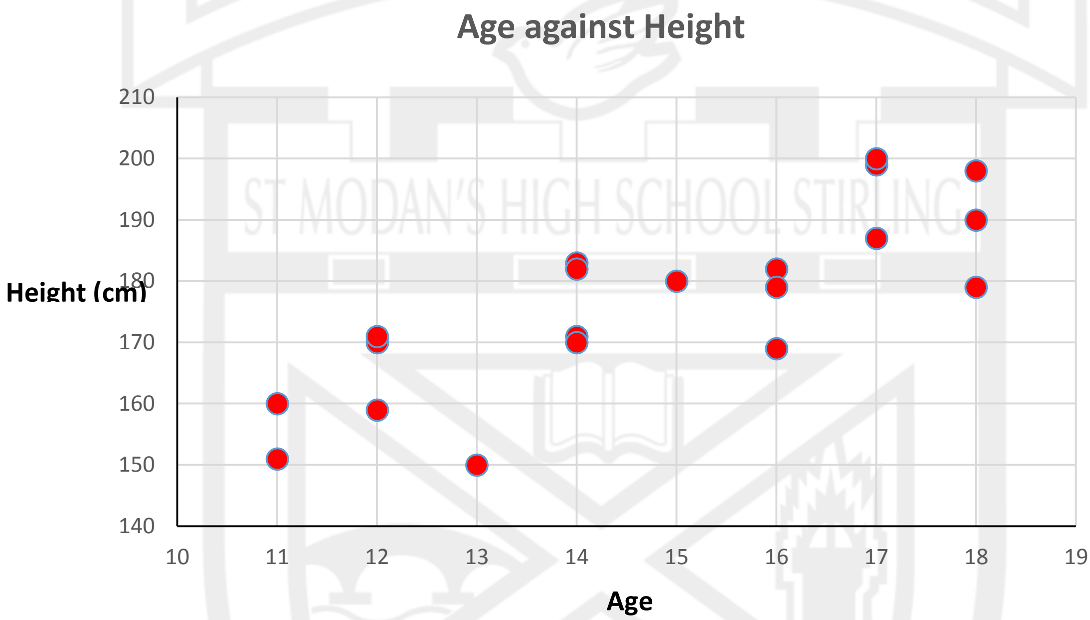
Now we can draw the pie chart.

Example 4 (scatter graph):

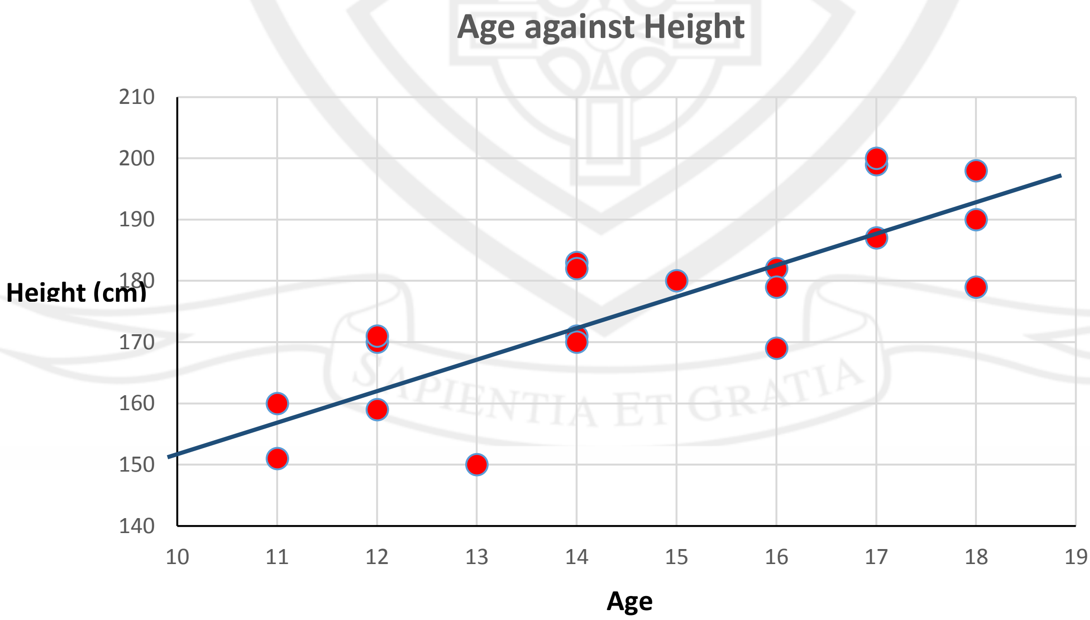
The table shows the age and height of a group of St Modan's pupils. Plot the data on a scatter graph.

Pupil	A	B	C	D	E	F	G	H	I	J
Age	16	17	14	17	14	12	12	16	18	15
Height (cm)	182	199	171	200	183	159	170	179	198	180

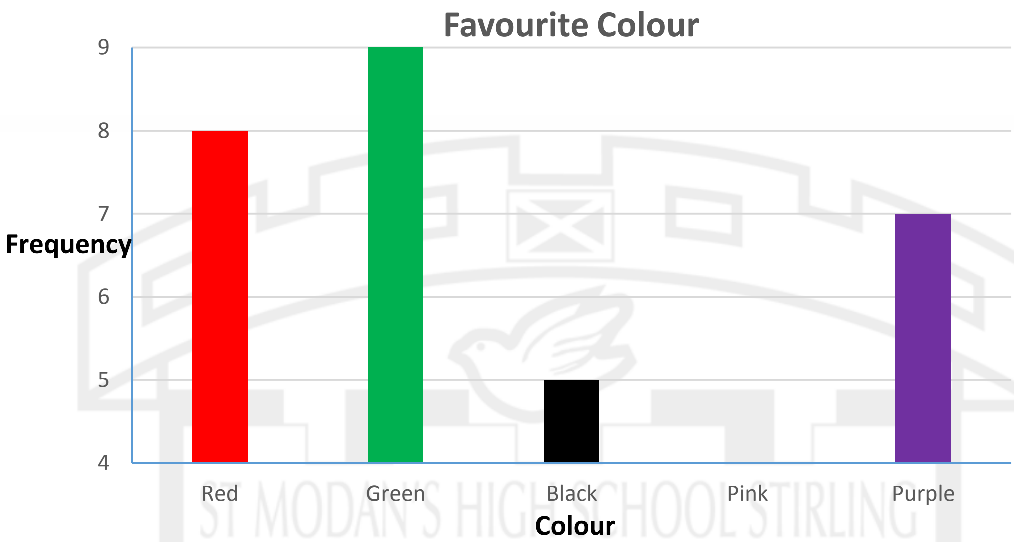
Pupil	K	L	M	N	O	P	Q	R	S	T
Age	18	18	17	16	11	11	13	12	14	14
Height (cm)	190	179	187	169	160	151	150	171	170	182

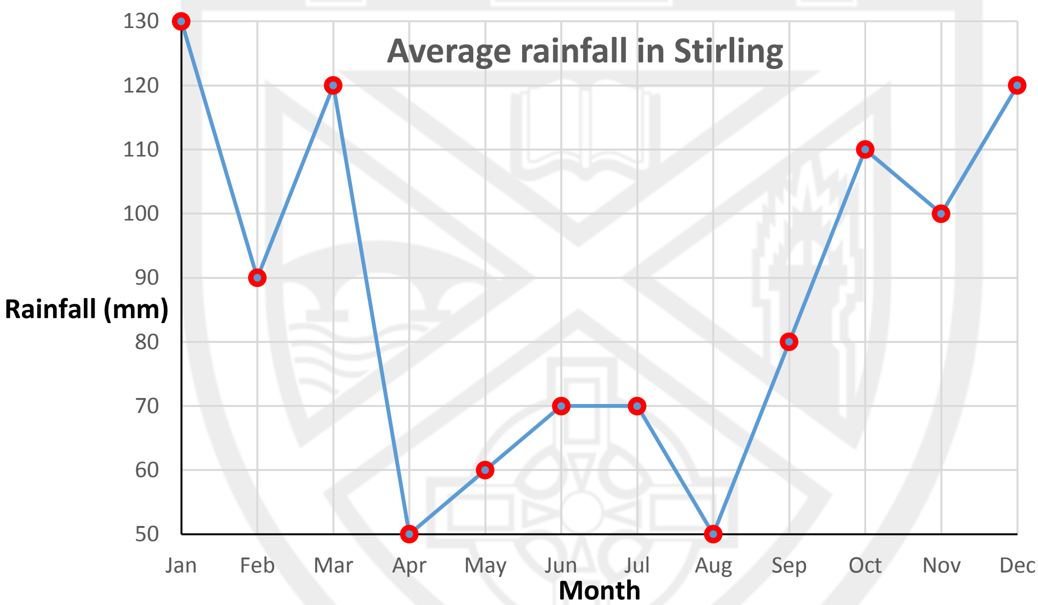


The graph above shows a positive trend, when trends like this occur it is common place to draw on a trendline (line of best fit) as below.



Example 5 (misleading graphs):
Graphs and charts can sometimes be somewhat misleading. Take the bar chart and line graphs from the examples above, drawn as they are below can make the data seem vastly different from how it really is.





Practice Questions

1. When asked how many hours of television they watched on a Saturday, a group of children gave the following responses

3	2	4	3	4	5	4	5	3	5
2	1	3	2	2	3	2	2	2	2
1	5	1	2	2	4	3	5	3	4

Show these results in a frequency table and draw a bar chart.

2. The table shows a patient's temperature, in ^{o}C, taken at 2-hourly intervals for a 24 hour period.

Time	12am	2am	4am	6am	8am	10am	12pm	2pm	4pm	6pm	8pm	10pm	12am
Temperature (^{o}C)	38.0	38.2	37.8	37.8	37.5	37.4	37.4	37.6	36.8	37.0	37.1	37.0	37.2

Draw a line graph to show the temperature over 24 hours.

3. A group of primary school children were asked about the musical instrument that they played. The results are shown in the frequency table.

Instrument	**Frequency**
Guitar	7
Piano	5
Recorder	12
Violin	4
None	8
Total	*36*

Use this to help you construct a pie chart of the pupils' use of musical instruments.

12am	2am	4am	6am	8am	10am	12pm	2pm	4pm	6pm	8pm	10pm
38.0	38.2	37.8	37.8	37.5	37.4	37.4	37.6	36.8	37.0	37.1	37.0

4. The table below shows the heights and weights of 20 patients in the hospital.

Use the information to construct a scatter graph.

Pupil	Height(cm)	Weight (kg)
A	182	71
B	199	78
C	171	65
D	200	70
E	183	60
F	159	55
G	170	50
H	179	66
I	198	76
J	180	63
K	190	68
L	179	75
M	187	72
N	169	76
O	160	49
P	151	41
Q	150	48
R	171	53
S	170	58
T	182	67

Ideas of Chance and Uncertainty

	Benchmarks
Second Level	• Uses the language of probability accurately to describe the likelihood of simple events occurring, for example equal chance; fifty-fifty; one in two, two in three; percentage chance; and $\frac{1}{6}$. • Plans and carries out simple experiments involving chance with repeated trials, for example, 'what is the probability of throwing a six if you throw a die fifty times?'. • Uses data to predict the outcome of a simple experiment.
Third Level	• Uses the probability scale of 0 to 1 showing probability as a fraction or decimal fraction. • Demonstrates understanding of the relationship between the frequency of an event happening and the probability of it happening. • Uses a given probability to calculate an expected outcome, for example, 'the probability of rain in June is 0·25 so how many days do we expect it to rain?'. • Calculates the probability of a simple event happening, for example, 'what is the probability of throwing a prime number on a 12 sided die?'. • Identifies all of the mutually exclusive outcomes of a single event and calculates the probability of each. • Investigates real-life situations which involve making decisions on the likelihood of events occurring and the consequences involved.
Fourth Level	• Calculates the probability and determines the expected occurrence of an event. • Applies knowledge and skills in calculating probability to make predictions.

Probability and Chance

The concept of probability is as important as it is misunderstood. It is vital to have an understanding of the nature of chance and variation in life. One area in which this is extremely important is in understanding risk and relative risk; weighing up options before making decisions.

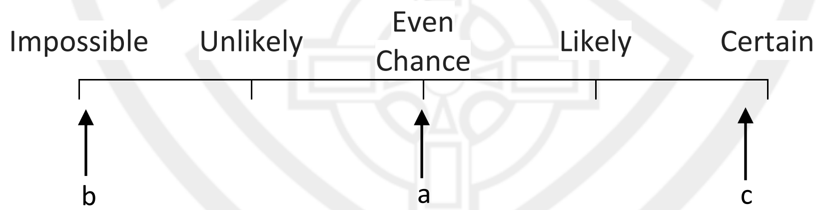
Probabilities can be defined in several ways; generally by using particular words to describe the chance of something happening, or mathematically by fractions, decimals, percentages or ratios.

The probability line

Impossible	Unlikely	Even Chance	Likely	Certain
$\frac{0}{1}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{1}$
0%	25%	50%	75%	100%
0	0.25	0.5	0.75	1

Example 1:
Mark on the probability line below the positions at which you think each of the statements would sit.

a. The probability of throwing a tail on a coin
b. The chances of winning the lottery
c. The probability that it will rain in Stirling at some point in December



Probability Calculations

Probability calculations are generally calculated as fractions in the first instance; they can then be converted to decimals and percentages as required.

We use a single simple formula for the calculation:

$$\boldsymbol{Probability\ of\ event = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}}$$

$$\boldsymbol{P(event) = \frac{favourable}{total}}$$

Example 1:
A variety bag of sweets contains 5 Mars Bars, 4 Milky Ways, 6 Bounty Bars and 5 Snicker Bars.
Anna chooses a sweet at random, calculate the probability that it is:

a. A Mars Bar

$$P(Mars) = \frac{favourable}{total}$$
$$P(Mars) = \frac{5}{20}$$
$$P(Mars) = \frac{1}{4}$$

b. Not a Bounty

$$P(Not\ Bounty) = \frac{favourable}{total}$$
$$P(Not\ Bounty) = \frac{14}{20}$$
$$P(Not\ Bounty) = \frac{7}{10}$$

Example 2:
A survey of the pupils in a class is taken, the eye colours were recorded and the table below shows the results:

Colour	Frequency
Green	7
Brown	18
Blue	5

a. Calculate the probability that a pupil has blue eyes.

$$P(blue) = \frac{favourable}{total}$$
$$P(blue) = \frac{5}{30}$$
$$P(blue) = \frac{1}{6}$$

b. The school has a role of 960 pupils. How many would you expect to have blue eyes?

$$\frac{1}{6}\ of\ 960$$
$$= 160$$

We would expect 160 pupils to have blue eyes.

Practice Questions

1. A die is rolled. Calculate the probability that the result will be

(a) a 2

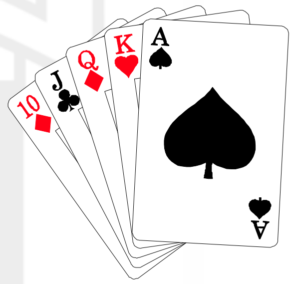
(b) a score greater than 3

(c) an odd number

2. A letter is chosen from the word INTERMEDIATE. Find the probability that it will be

(a) a vowel **(b)** a T **(c)** an E **(d)** an M

3. A card is drawn from a deck of 52 playing cards.
Find the probability that it will be



(a) a club **(b)** a red card

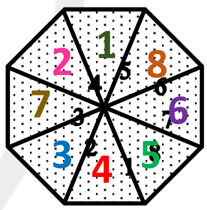
(c) an Ace **(d)** a face card

(e) 3 of spades **(f)** a black king

4. A bag contains 3 red discs, 5 blue discs and 2 green discs. A disc is chosen at random from the bag. Find the probability that it is

(a) blue **(b)** red **(c)** green **(d)** not red

5. This spinner with numbers 1-8 is used in a game.



What is the probability of getting

(a) a 6 **(b)** an even number

(c) a number greater than 5 **(d)** a multiple of 3

(e) a factor of 8 **(f)** a number less than 3?

Solutions to Practice Questions

Rounding

1	**(a)**	3	**(b)**	6	**(c)**	1	**(d)**	8
2	**(a)**	12	**(b)**	35	**(c)**	28	**(d)**	83
3	**(a)**	20	**(b)**	70	**(c)**	70	**(d)**	90
4	**(a)**	210	**(b)**	1280	**(c)**	7620	**(d)**	340
5	**(a)**	300	**(b)**	200	**(c)**	3400	**(d)**	83100
6	**(a)**	0.3	**(b)**	0.3	**(c)**	0.6	**(d)**	0.6
7	**(a)**	2.9	**(b)**	5.7	**(c)**	1.5	**(d)**	8.3
8	**(a)**	62.04	**(b)**	15.62	**(c)**	31.48	**(d)**	18.30
9	**(a)**	2.92	**(b)**	5.66	**(c)**	1.41	**(d)**	8.32
10	**(a)**		**(b)**		**(c)**		**(d)**	

Significant Figures

1	**(a)**	20	**(b)**	6	**(c)**	80	**(d)**	30
	(e)	100	**(f)**	300	**(g)**	300	**(h)**	800
2	**(a)**	11	**(b)**	560	**(c)**	4.0	**(d)**	12000
	(e)	1.0	**(f)**	93	**(g)**	56	**(h)**	610
3	**(a)**	9	**(b)**	90	**(c)**	0.2	**(d)**	700
	(e)	2	**(f)**	6	**(g)**	30	**(h)**	30
4	**(a)**	24	**(b)**	19	**(c)**	6400	**(d)**	5.0
	(e)	0.053	**(f)**	0.0061	**(g)**	0.087	**(h)**	14000

Adding

1	**(a)**	113	**(b)**	144	**(c)**	184	**(d)**	234
	(e)	194	**(f)**	167	**(g)**	337	**(h)**	265
2	**(a)**	221	**(b)**	144	**(c)**	192	**(d)**	182
	(e)	169	**(f)**	243	**(g)**	271	**(h)**	246
3	**(a)**	240	**(b)**	323	**(c)**	362	**(d)**	420
	(e)	369	**(f)**	259	**(g)**	445	**(h)**	518
4	**(a)**	17	**(b)**	17.4	**(c)**	13.6	**(d)**	12.5
	(e)	19.1	**(f)**	21.3	**(g)**	28.2	**(h)**	22.8
5	**(a)**	20.8	**(b)**	19.8	**(c)**	18.6	**(d)**	20
	(e)	20.3	**(f)**	18.6	**(g)**	45.7	**(h)**	42.3
6	**(a)**	22.8	**(b)**	35.8	**(c)**	28.7	**(d)**	35.8
	(e)	24.9	**(f)**	32.8	**(g)**	51.8	**(h)**	65.6

Subtracting

1	**(a)**	19	**(b)**	8	**(c)**	0	**(d)**	34
	(e)	61	**(f)**	63	**(g)**	139	**(h)**	151
2	**(a)**	28	**(b)**	49	**(c)**	1	**(d)**	143
	(e)	84	**(f)**	31	**(g)**	162	**(h)**	219
3	**(a)**	115	**(b)**	114	**(c)**	258	**(d)**	152
	(e)	295	**(f)**	94	**(g)**	62	**(h)**	161
4	**(a)**	2.5	**(b)**	1.1	**(c)**	7.7	**(d)**	8.2
	(e)	5.6	**(f)**	6.9	**(g)**	8	**(h)**	12.1
5	**(a)**	0.2	**(b)**	3	**(c)**	10	**(d)**	9.7
	(e)	1	**(f)**	14	**(g)**	10	**(h)**	12.5
6	**(a)**	14	**(b)**	17	**(c)**	8	**(d)**	20
	(e)	3.2	**(f)**	32	**(g)**	39	**(h)**	6

Multiplication

1	**(a)**	260	**(b)**	696	**(c)**	864	**(d)**	810
	(e)	196	**(f)**	568	**(g)**	560	**(h)**	350
2	**(a)**	5850	**(b)**	4524	**(c)**	5088	**(d)**	8064
	(e)	6912	**(f)**	7178	**(g)**	3360	**(h)**	5856
3	**(a)**	4929	**(b)**	31691	**(c)**	59228	**(d)**	18785
	(e)	5060	**(f)**	25275	**(g)**	3627	**(h)**	28954
4	**(a)**	8.1	**(b)**	3.06	**(c)**	9.7	**(d)**	4.2
	(e)	5.53	**(f)**	2.16	**(g)**	3.6	**(h)**	5.2
5	**(a)**	60.06	**(b)**	34.81	**(c)**	70.56	**(d)**	45.9
	(e)	53.29	**(f)**	58.28	**(g)**	64.74	**(h)**	36
6	**(a)**	9.333	**(b)**	30.051	**(c)**	57.855	**(d)**	15.717
	(e)	81.88	**(f)**	1.998	**(g)**	18.525	**(h)**	53.975

Division

1	**(a)**	15	**(b)**	18	**(c)**	21	**(d)**	27
	(e)	10	**(f)**	30	**(g)**	27	**(h)**	23
2	**(a)**	15	**(b)**	15	**(c)**	16	**(d)**	29
	(e)	17	**(f)**	27	**(g)**	23	**(h)**	15
3	**(a)**	2.1	**(b)**	1.6	**(c)**	2	**(d)**	2.9
	(e)	1.5	**(f)**	1.5	**(g)**	2.7	**(h)**	1.2
4	**(a)**	0.18	**(b)**	0.1	**(c)**	0.19	**(d)**	0.1
	(e)	0.2	**(f)**	0.11	**(g)**	0.28	**(h)**	0.16
5	**(a)**	15	**(b)**	18	**(c)**	21	**(d)**	27
	(e)	10	**(f)**	30	**(g)**	27	**(h)**	23

Integers – Addition and Subtraction

1	**(a)**	18	**(b)**	26	**(c)**	18	**(d)**	23
	(e)	18	**(f)**	19	**(g)**	26	**(h)**	13
2	**(a)**	2	**(b)**	-6	**(c)**	4	**(d)**	-3
	(e)	8	**(f)**	5	**(g)**	6	**(h)**	2
3	**(a)**	-9	**(b)**	0	**(c)**	-14	**(d)**	1
	(e)	-25	**(f)**	-25	**(g)**	-18	**(h)**	-19
4	**(a)**	-5	**(b)**	2	**(c)**	-1	**(d)**	-7
	(e)	-32	**(f)**	-29	**(g)**	-26	**(h)**	-27
5	**(a)**	-9	**(b)**	-3	**(c)**	-3	**(d)**	-1
	(e)	-35	**(f)**	-23	**(g)**	-34	**(h)**	-28

Integers – Multiplication and Division

1	**(a)**	16	**(b)**	77	**(c)**	10	**(d)**	44
	(e)	-6	**(f)**	-14	**(g)**	-108	**(h)**	-36
2	**(a)**	-36	**(b)**	-54	**(c)**	-7	**(d)**	-21
	(e)	27	**(f)**	3	**(g)**	9	**(h)**	9
3	**(a)**	12	**(b)**	7	**(c)**	12	**(d)**	2
	(e)	-12	**(f)**	-9	**(g)**	-7	**(h)**	-11
4	**(a)**	-5	**(b)**	-13	**(c)**	-3	**(d)**	-6
	(e)	6	**(f)**	7	**(g)**	5	**(h)**	8

BODMAS

1	(a)	11	(b)	5	(c)	21	(d)	3
	(e)	19	(f)	12	(g)	34	(h)	56
2	(a)	16	(b)	48	(c)	5	(d)	55
	(e)	8	(f)	4	(g)	30	(h)	5
3	(a)	24	(b)	2	(c)	21	(d)	36
	(e)	8	(f)	64	(g)	33	(h)	6

Simplifying Fractions

1	(a)	$\frac{1}{6}$	(b)	$\frac{4}{15}$	(c)	$\frac{2}{3}$	(d)	$\frac{5}{9}$
	(e)	$\frac{9}{10}$	(f)	$\frac{2}{5}$	(g)	$\frac{2}{3}$	(h)	$\frac{1}{4}$
2	(a)	$\frac{1}{4}$	(b)	$\frac{4}{5}$	(c)	$\frac{2}{11}$	(d)	$\frac{17}{60}$
	(e)	$\frac{1}{5}$	(f)	$\frac{5}{22}$	(g)	$\frac{7}{18}$	(h)	$\frac{3}{10}$

Mixed and Improper Fractions

1	(a)	$1\frac{5}{7}$	(b)	$4\frac{2}{3}$	(c)	$7\frac{1}{2}$	(d)	$3\frac{1}{6}$
	(e)	$4\frac{2}{5}$	(f)	$4\frac{6}{7}$	(g)	$16\frac{3}{4}$	(h)	$9\frac{4}{9}$
2	(a)	$\frac{3}{2}$	(b)	$\frac{18}{5}$	(c)	$\frac{14}{3}$	(d)	$\frac{21}{8}$
	(e)	$\frac{41}{7}$	(f)	$\frac{59}{7}$	(g)	$\frac{49}{5}$	(h)	$\frac{44}{15}$

Multiplying Fractions

1	(a)	$\frac{1}{7}$	(b)	$\frac{1}{10}$	(c)	$2/7$	(d)	$\frac{1}{12}$
	(e)	$\frac{1}{20}$	(f)	$\frac{4}{7}$	(g)	$\frac{2}{7}$	(h)	$\frac{1}{14}$
2	(a)	$1\frac{2}{3}$	(b)	$2\frac{1}{12}$	(c)	$6\frac{1}{4}$	(d)	$2\frac{11}{12}$
	(e)	$7\frac{3}{20}$	(f)	$11\frac{5}{9}$	(g)	$5\frac{1}{6}$	(h)	$15\frac{3}{4}$

Dividing Fractions

1	(a)	$\frac{7}{16}$	(b)	$1\frac{1}{9}$	(c)	$\frac{7}{8}$	(d)	$5\frac{1}{3}$
	(e)	$12\frac{4}{5}$	(f)	$1\frac{2}{7}$	(g)	$\frac{2}{7}$	(h)	$1\frac{31}{32}$
2	(a)	$\frac{15}{16}$	(b)	$\frac{3}{4}$	(c)	1	(d)	$1\frac{1}{20}$
	(e)	$1\frac{21}{44}$	(f)	$1\frac{5}{8}$	(g)	$\frac{62}{75}$	(h)	$\frac{25}{28}$

Adding and Subtracting Fractions

1	(a)	$\frac{23}{28}$	(b)	$\frac{19}{30}$	(c)	$1\frac{1}{14}$	(d)	$\frac{19}{24}$
	(e)	$\frac{59}{80}$	(f)	$\frac{4}{21}$	(g)	$\frac{1}{8}$	(h)	$\frac{12}{65}$
2	(a)	$2\frac{7}{12}$	(b)	$2\frac{11}{12}$	(c)	5	(d)	$3\frac{5}{12}$
	(e)	$1\frac{1}{20}$	(f)	$1\frac{2}{3}$	(g)	$\frac{13}{30}$	(h)	$\frac{9}{20}$

Percentages

1	(a)	£50	(b)	£32	(c)	180kg	(d)	228g
	(e)	$9	(f)	£15	(g)	€ 135	(h)	63kg
2	(a)	176g	(b)	£193.20	(c)	316kg	(d)	124.5g
	(e)	$2.80	(f)	239.2g	(g)	€ 367.50	(h)	744ml

Best Buy

1	**(a)**	3 litre	**(b)**	550g	**(c)**	0.7 litre	**(d)**	0.8kg
	(e)	1.5kg						

Foreign Exchange

1	**(a)**	€282.66	**(b)**	8981.66 R	**(c)**	$22
2	**(a)**	£542.84	**(b)**	£307.91	**(c)**	£7.95

Time differences

1	**(a)**	30 min	**(b)**	1h 9 min	**(c)**	1h 47 min	**(d)**	1h 48 min
	(e)	3h 49 min	**(f)**	17h 29 min	**(g)**	54 days	**(h)**	77 days
	(i)	131 days						

Fractional and decimal time

1	**(a)**	0·60 h	**(b)**	0·95 h	**(c)**	0·33 h	**(d)**	1·30 h
	(e)	2·10 h	**(f)**	5·80 h	**(g)**	2·35 h	**(h)**	9·92 h
	(i)	22·13 h						

Speed distance and time

1	**(a)**	200 km	**(b)**	390 km	**(c)**	217·5 km	**(d)**	0·2 km
2	**(a)**	73·91 km/h	**(b)**	392 km/h	**(c)**	72·5 km/h	**(d)**	9·45 km/h
	(e)	11·36 km/h						
3	**(a)**	5h 19 min	**(b)**	11h 14 min	**(c)**	1h 12 min	**(d)**	10h 20 min
	(e)	1h 36 min						

Conversions

1	**(a)**	$250mm$	**(b)**	$230cm$	**(c)**	$4800m$	**(d)**	$2.3cm$
	(e)	$0.47km$	**(f)**	$3.05m$	**(g)**	$0.03m$	**(h)**	$0.0024km$
2	**(a)**	$2000g$	**(b)**	$3400kg$	**(c)**	$0.4kg$	**(d)**	$0.6g$
	(e)	$0.23T$	**(f)**	$35\,000mg$	**(g)**	$0.46T$	**(h)**	$0.0056T$
3	**(a)**	$24ml$	**(b)**	$2300ml$	**(c)**	$2.5l$	**(d)**	$5000cm^3$
	(e)	$6l$	**(f)**	$3800ml$	**(g)**	$0.025l$	**(h)**	$0.095l$

Area and Volume Calculations

1	**(a)**	$60mm^2$	**(b)**	$38.5cm^2$	**(c)**	$615.8m^2$	**(d)**	$14cm^2$
	(e)	$3675m^2$	**(f)**	$1777.1cm^2$				
2	**(a)**	$616m^3$	**(b)**	$210cm^3$	**(c)**	$150.8mm^3$		

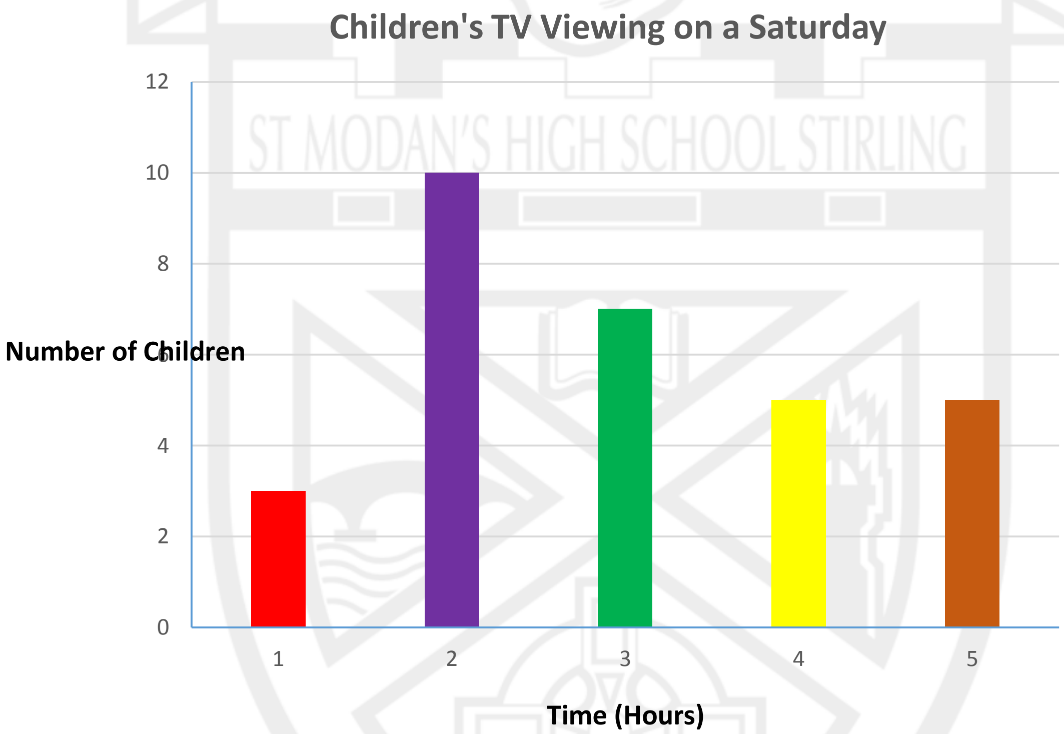
Averages

		MEAN	**MEDIAN**		**MODE**	**RANGE**
1	**(a)**	7	7		7	8
	(b)	6	5		3	14
	(c)	2.3	2.2		2.2	1.8
	(d)	84.6	85.5		81	10
	(e)	2.5	2		2	3
	(f)	1.2	1		0.8	1.6
	(g)	323	322		308	35
	(h)	13.2	12.15		9.7	8.3
2	**(a)**	MEDIAN- 3	RANGE- 4	**(b)**	MODE- 3	

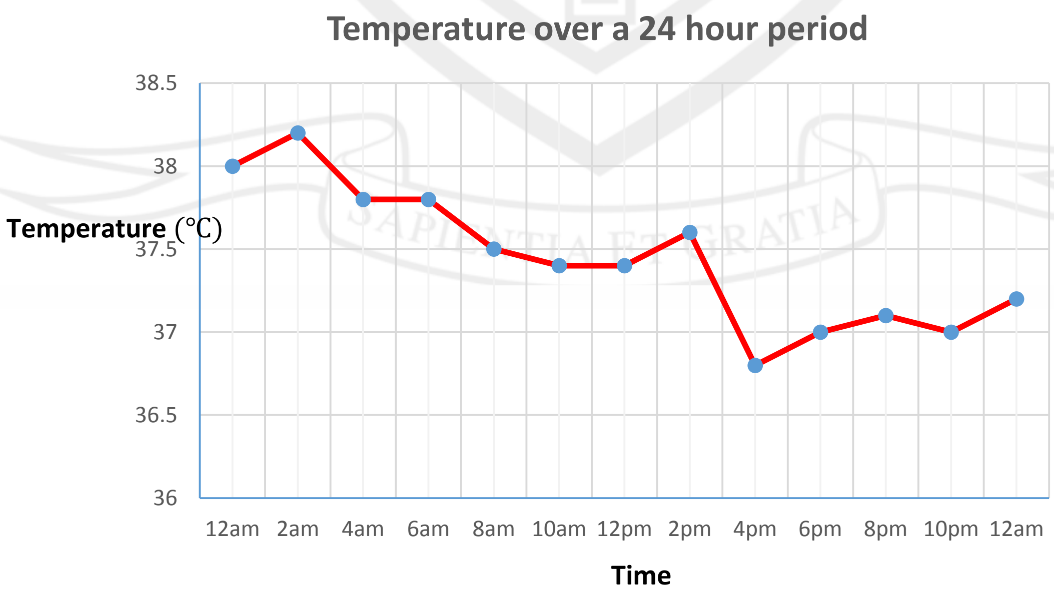
Graphical Displays

1.

Time watching television (hours)	Frequency
1	3
2	10
3	7
4	5
5	5
Total	*30*

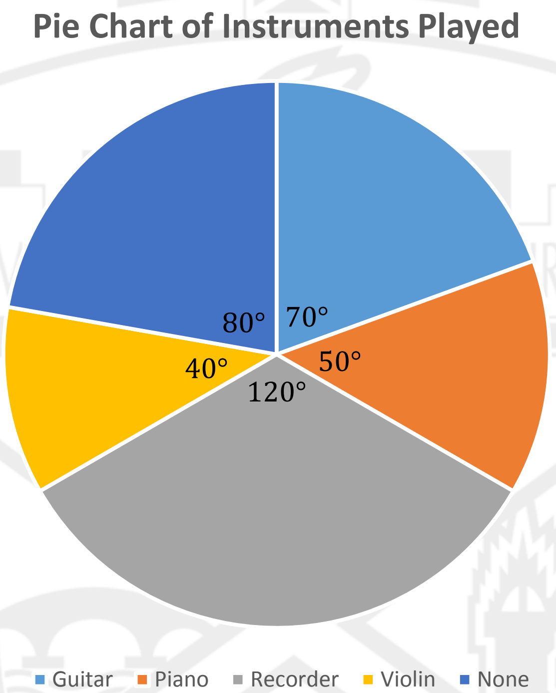


2.

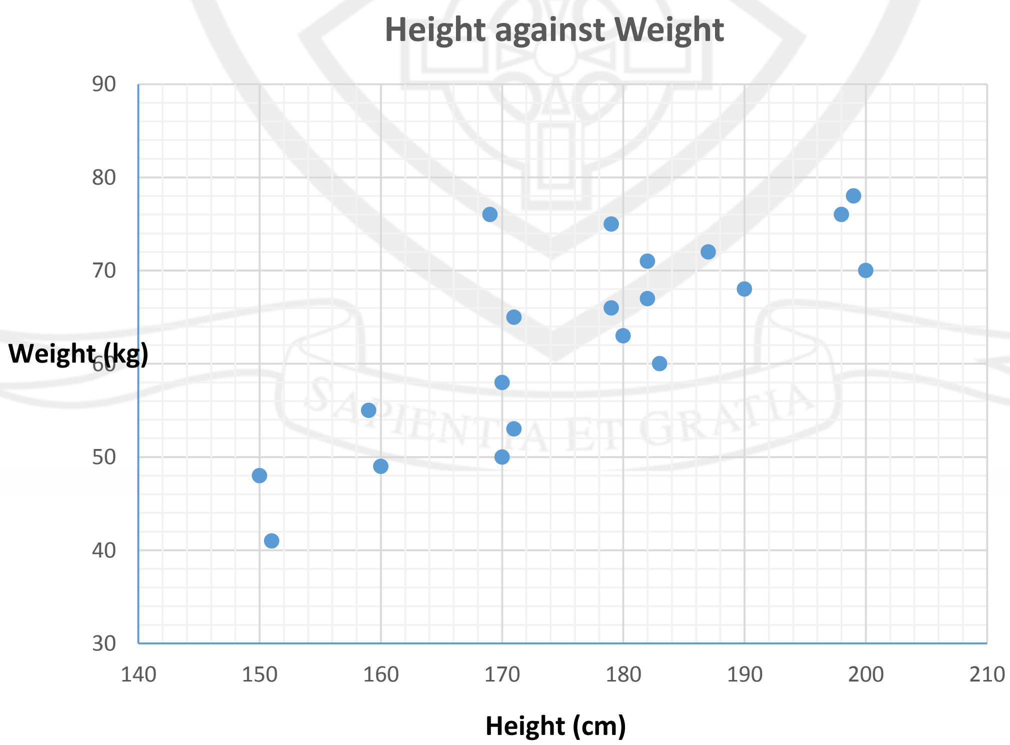


3.

Instrument	Frequency	Angle
Guitar	7	70°
Piano	5	50°
Recorder	12	120°
Violin	4	40°
None	8	80°
Total	*36*	360°



4.



Probability

1	**(a)**	$\frac{1}{6}$	**(b)**	$\frac{1}{2}$	**(c)**	$\frac{1}{2}$		
2	**(a)**	$\frac{1}{2}$	**(b)**	$\frac{1}{6}$	**(c)**	$\frac{1}{4}$	**(d)**	$\frac{1}{12}$
3	**(a)**	$\frac{1}{4}$	**(b)**	$\frac{1}{2}$	**(c)**	$\frac{1}{13}$	**(d)**	$\frac{3}{13}$
	(e)	$\frac{1}{52}$	**(f)**	$\frac{1}{26}$				
4	**(a)**	$\frac{1}{2}$	**(b)**	$\frac{3}{10}$	**(c)**	$\frac{1}{5}$	**(d)**	$\frac{7}{10}$
5	**(a)**	$\frac{1}{8}$	**(b)**	$\frac{1}{2}$	**(c)**	$\frac{3}{8}$	**(d)**	$\frac{1}{4}$
	(e)	$\frac{1}{2}$	**(f)**	$\frac{1}{4}$				